15.1 Homework

(9) $\iint_R (15-3x)\,dA$ where $R=[0,5]\times[0,3]$

$\rightarrow \int_0^5 (15-3x)\,dx = \left[15x - \frac{3x^2}{2}\right]\Big|_0^5 = \left(75 - \frac{75}{2}\right) = \frac{150}{2} - \frac{75}{2} = \frac{75}{2}$

$\rightarrow \int_0^3 \frac{75}{2}\,dy = \frac{75}{2}y\Big|_0^3 = \boxed{\frac{225}{2}}$

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(15) $\iint_R x^3\,dA$, $R=[-4,4]\times[0,5]$

$\rightarrow \int_{-4}^4 x^3\,dx = \frac{x^4}{4}\Big|_{-4}^4 = 0$

$\rightarrow \int_0^5 0\,dy = \boxed{0}$

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(21) $\int_4^9\int_{-3}^8 1\,dx\,dy$

$\rightarrow \int_{-3}^8 1\,dx = x\Big|_{-3}^8 = 8+3 = 11$

$\rightarrow \int_4^9 11\,dy = 11y\Big|_4^9 = 99-44 = \boxed{55}$

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(23) $\int_{-1}^1\int_0^\pi x^2 \sin y\,dy\,dx$

$\rightarrow \int_0^\pi x^2 \sin y\,dy = -x^2\cos y\Big|_0^\pi = -x^2(-1-1) = 2x^2$

$\rightarrow \int_{-1}^1 2x^2\,dx = \frac{2x^3}{3}\Big|_{-1}^1 = \frac{2}{3} + \frac{2}{3} = \boxed{\frac{4}{3}}$

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(25) $\int_2^6\int_1^4 x^2\,dx\,dy$

$\rightarrow \int_1^4 x^2\,dx = \frac{x^3}{3}\Big|_1^4 = \frac{64}{3} - \frac{1}{3} = \frac{63}{3}$

$\rightarrow \int_2^6 \frac{63}{3}\,dy = \frac{63}{3}y\Big|_2^6 = \boxed{84}$

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(31) $\int_1^2\int_0^4 \frac{1}{x+y}\,dy\,dx$

$\rightarrow \int_0^4 \frac{1}{x+y}\,dy = \ln(x+y)\Big|_0^4 = \ln(x+4) - \ln(x)$

$\rightarrow \int_1^2 [\ln(x+4) - \ln(x)]\,dx = \left[(x+4)\ln(x+4) - (x+4) - x\ln(x) + x\right]_1^2$

$\rightarrow (6\ln(6) - 6 - 2\ln(2) + 2) - (5\ln(5) - 5 - 0 + 1) = \boxed{6\ln(6) - 2\ln(2) - 5\ln(5)}$

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(33) $\int_0^4\int_0^5 \frac{1}{\sqrt{x+y}}\,dy\,dx$

$\rightarrow \int_0^5 \frac{dy}{\sqrt{x+y}} = 2\sqrt{x+y}\Big|_0^5 = 2\left(\sqrt{x+5} - \sqrt{x}\right) = 2\sqrt{x+5} - 2\sqrt{x}$

$\rightarrow \int_0^4 \left(2\sqrt{x+5} - 2\sqrt{x}\right)dx = \boxed{\frac{4}{3}\left(19 - 5\sqrt{5}\right)}$

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(35) $\int_1^2\int_1^3 \frac{\ln(xy)}{y}\,dy\,dx$

$\rightarrow \int_1^3 \frac{\ln(xy)}{y}\,dy = \frac{x(\ln(xy)-1)}{y}\Big|_1^3 = \frac{\ln^2(xy)}{2}\Big|_1^3 = \frac{\ln^2(3x)}{2} - \frac{\ln^2(x)}{2}$

$\rightarrow \int_1^2\left(\frac{\ln^2(3x)}{2} - \frac{\ln^2(x)}{2}\right)dx = \boxed{\frac{1}{2}(\ln(3))(\ln(48)-2)}$

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(37) $\iint_R \frac{x}{y}\,dA, \quad R=[-2,4]\times[1,3]$

$\rightarrow \int_{-2}^4 \frac{x}{y}\,dx = \frac{x^2}{2y}\Big|_{-2}^4 = \frac{16}{2y} - \frac{4}{2y} = \frac{8}{y} - \frac{2}{y}$

$\rightarrow \int_1^3\left(\frac{8}{y} - \frac{2}{y}\right)dy = \boxed{6\ln(3)}$

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(41) $\iint_R e^x \sin y\,dA, \quad R=[0,2]\times[0,\frac{\pi}{4}]$

$\rightarrow \int_0^2 e^x \sin y\,dx = xe^x \sin y\Big|_0^2 = 2e^2 \sin y$

$\rightarrow \int_0^{\pi/4} 2e^2 \sin y\,dy = -2e^2\cos y\Big|_0^{\pi/4} = -2e^2\left(\frac{\sqrt{2}}{2} - 1\right) = \boxed{(e^2-1)\left(1-\frac{\sqrt{2}}{2}\right)}$

15.2 Homework

③ $y = 1 - x^2$

$\rightarrow$ Vertically simple region: $0 \le x \le 1,\ 0 \le y \le 1 - x^2$

$\rightarrow$ Horizontally simple region: $0 \le y \le 1,\ 0 \le x \le \sqrt{1-y}$

$\rightarrow \int_0^1 \int_0^{1-x^2} xy\, dy\, dx = \boxed{\frac{1}{12}}$

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⑤ $(0, 2)$ and $(4, 0)$ $\Rightarrow \frac{0-2}{4-0} = \frac{-2}{4} = -\frac{1}{2}$

$\rightarrow y - 2 = -\frac{1}{2}(x - 0) \Rightarrow y - 2 = -\frac{1}{2}x \Rightarrow y = -\frac{1}{2}x + 2$

$\rightarrow$ Domain: $0 \le x \le 4$ and $-\frac{1}{2}x + 2 \le y \le 2$

$\rightarrow \int_0^4 \int_{-\frac{1}{2}x+2}^{2} (x^2 y)\, dy\, dx = \boxed{38.4}$

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⑥ Same area as above so $\boxed{38.4}$

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⑦ Domain: $0 \le y \le 2$ and $y \le x \le 4$

$\rightarrow \int_0^2 \int_y^4 (x^2 y)\, dx\, dy = \boxed{40.53}$

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⑪ $D = 0 \le y \le \sqrt{4 - x^2}$ and $1 \le x \le 2$

$\rightarrow \int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\, dy\, dx = \boxed{-\frac{3}{4} + \ln(4)}$

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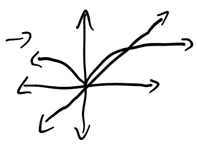
⑲ $f(x,y) = x;\ 0 \le x \le 1,\ 1 \le y \le e^{x^2}$

$\rightarrow \int_0^1 \int_1^{e^{x^2}} x\, dy\, dx$

$\rightarrow \int_1^{e^{x^2}} x\, dy = xy \Big|_1^{e^{x^2}} = x\left[e^{x^2} - 1\right] = xe^{x^2} - x$

$\rightarrow \int_0^1 (xe^{x^2} - x)\, dx = \boxed{\frac{e-2}{2}}$

㉑ $f(x,y) = 2xy;$ bounded by $x = y,\ x = y^2$



$\rightarrow D = 0 \le x \le 1$ and $x \le y \le \sqrt{x}$

$\rightarrow \int_0^1 \int_x^{\sqrt{x}} (2xy)\, dy\, dx = \boxed{\frac{1}{12}}$

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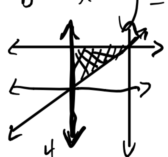
(25) $\int_0^4 \int_x^4 f(x,y)\, dy\, dx$

$\rightarrow D = x \le y \le 4$ and $0 \le x \le 4$

$\rightarrow$ 

$\rightarrow \int_0^4 \int_0^y f(x,y)\, dx\, dy$

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(31) $f(x,y) = (\ln y)^{-1}$; $y = e^x$ and $y = e^{\sqrt{x}}$

$\rightarrow \int_0^1 \int_{e^x}^{e^{\sqrt{x}}} (\ln y)^{-1}\, dy\, dx = \boxed{0.718}$

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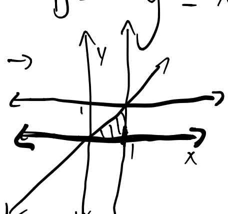
(33) $\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$

$\rightarrow D = y \le x \le 1$ and $0 \le y \le 1$

$\rightarrow$ 

$\rightarrow \int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx = \boxed{1 - \cos 1}$

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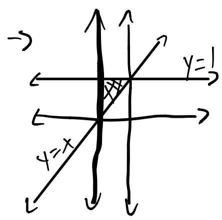
(35) $\int_0^1 \int_x^1 x e^{y^3}\, dy\, dx$

$\rightarrow D = x \le y \le 1$ and $0 \le x \le 1$



$\rightarrow \int_0^1 \int_0^y x e^{y^3} \, dx \, dy = \boxed{\frac{e-1}{6}}$

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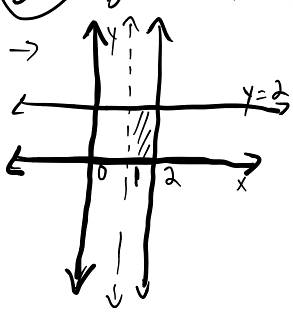
(37) $D = 0 \le x \le 2, \quad 0 \le y \le 2, \quad x > 1 \; ; \; \iint_D e^{x+y} \, dA$



$\rightarrow \iint_D (e^{x+y}) \, dA = \boxed{e^4 - 3e^2 + 2e}$

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(43) $f(x,y) = \frac{\sin y}{y}$

$\rightarrow \int_1^2 \int_y^{2y} \frac{\sin y}{y} \, dx \, dy = \boxed{0.956}$

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(49) $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$

$\rightarrow \boxed{\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \left[(8 - x^2 - y^2) - (x^2 + y^2) \right] dy \, dx}$
