15.1 HW - 9, 15, 21, 23, 25, 31, 33, 35, 37, 41

9. $\iint_R (15 - 3x)\, dA \quad R = [0,5] \times [0,3]$

$0 \le x \le 5, \quad 0 \le y \le 3$

$\int_0^3 \int_0^5 (15-3x)\, dx\, dy$

→ inner integral:

$\int_0^5 (15-3x)\, dx = \left[15x - \frac{3}{2}x^2\right]\Big|_0^5$

$= \left[15(5) - \frac{3}{2}(5)^2\right] - 0 = 75 - \frac{3(25)}{2} = 37.5 = \frac{75}{2}$

→ outer integral:

$\int_0^3 \left(\frac{75}{2}\right) dy = \left(\frac{75}{2}y\right)\Big|_0^3 = \frac{75(3)}{2} - 0 = \frac{225}{2}$

15. $\iint_R x^3\, dA \quad R = [-4,4] \times [0,5]$

$-4 \le x \le 4 \quad 0 \le y \le 5$

$\int_0^5 \int_{-4}^4 (x^3)\, dx\, dy$

→ inner integral: $\int_{-4}^4 (x^3)\, dx = \left[\frac{x^4}{4}\right]\Big|_{-4}^4$

$= \left(\frac{4^4}{4}\right) - \left(\frac{(-4)^4}{4}\right)$

$= \frac{256}{4} - \frac{256}{4} = 0$

→ outer integral

$\int_0^5 (0)\, dy = 0$

→ $\iint_R x^3\, dA \quad R = [-4,4] \times [0,5] = 0$

21. $\int_4^9 \int_{-3}^8 1\, dx\, dy$ → inner integral: $\int_{-3}^8 1\, dx = [x]\Big|_{-3}^8 = 8 - (-3) = 11$

→ outer integral:

$\int_4^9 11\, dy = [11y]\Big|_4^9 = 9(11) - 4(11) = 99 - 44 = 55$

23. $\int_{-1}^1 \int_0^\pi x^2 \sin y\, dy\, dx$ → inner integral: $\int_0^\pi x^2 \sin(y)\, dy$

$= \left[x^2 \cos(y)\right]\Big|_0^\pi = x^2\cos(\pi) - x^2\cos(0)$

$= -x^2 - x^2 = -2x^2$

→ outer integral: $\int_{-1}^1 (-2x^2) = \left[-\frac{2}{3}x^3\right]\Big|_{-1}^1$

$= \frac{-2}{3} - \frac{2}{3} = \frac{-4}{3}$

25. $\int_2^6 \int_1^4 x^2 \, dx \, dy$ → inner integral. $\int_1^4 (x^2) dx = \left[\frac{x^3}{3}\right]\Big|_1^4 = \left(\frac{4^3}{3}\right) - \left(\frac{1^3}{3}\right)$

$= \frac{64}{3} - \frac{1}{3} = \frac{63}{3} = 33$

→ outer integral

$\int_2^6 (33) dy = (33y)\Big|_2^6 = (33)(6) - (33)(2) = 198 - 66 = 132$

31. $\int_1^2 \int_0^4 \left(\frac{1}{x+y}\right) dy\,dx$ → inner integral. $\int_0^4 \left(\frac{1}{x+y}\right) dy = \ln(x+y)\Big|_0^4$

$\ln(x+4) - \ln(x)$

outer integral:

$\int_1^2 \ln\left(1 + \frac{4}{x}\right) dx$

integration by parts

$u = \ln\left(\frac{x+4}{x}\right)$ $dv = dx$

$du = \frac{\quad}{1+4x}$

33.

$\int_0^4 \int_0^5 \frac{1}{\sqrt{x+y}} dy\,dx$ → inner integral: $\int_0^5 (x+y)^{-1/2} dy$

$= 2(x+y)^{1/2}\Big|_0^5 =$

outer integral:

$2\sqrt{x+5} - 2\sqrt{x}$

$\int_0^4 \left(2\sqrt{x+5} - 2\sqrt{x}\right) dx$

$= \int_0^4 2(x+5)^{1/2} + 2x^{1/2} dx = \frac{4}{3}(x+5)^{3/2} + \frac{4}{3}x^{3/2}$

$= \left[\frac{4}{3}(4+5)^{3/2} + \frac{4}{3}(4)^{3/2}\right] - \left[\frac{4}{3}(0+5)^{3/2} + \frac{4}{3}(0)^{3/2}\right]$

35. $\int_1^2 \int_1^3 \frac{\ln(xy)}{y} dy\,dx$ → inner integral: $\int_1^3 \frac{\ln(xy)}{y} dy = \int_1^3 u\,du$

$u = \ln(xy)$ $\quad = \frac{u^2}{2}\Big|_1^3$

$du = \frac{1}{y}$

$= \frac{[\ln(3x)]^2}{2} - \frac{[\ln(x)]^2}{2}$

$\int_1^2 \frac{\ln^2(3x) - \ln^2(x)}{2} dx = \frac{1}{2}\int_1^2 \ln^2(3x)$

$- \frac{1}{2}\int_1^2 \ln^2(x)$

31. $\iint_R \frac{x}{y} dA \quad R = [-2,4] \times [1,3]$

$\int_1^3 \int_{-2}^4 \frac{x}{y} \, dx\,dy \rightarrow$ inner int: $\int_{-2}^4 x(y^{-1})\,dx = \frac{x^2}{2y}\Big|_{-2}^4 = \frac{16}{2y} - \frac{4}{2y} = \frac{12}{2y} = 6y$

$\rightarrow$ outer int: $\int_1^3 6y\,dy = 3y^2\Big|_1^3 = 3(3)^2 - (3)(1)^2 = 27 - 3$
$= 24$

41. $\iint_R e^x \sin(y)\,dA \quad R = [0,2] \times [0, \frac{\pi}{4}]$

$\int_0^{\frac{\pi}{4}} \int_0^2 e^x \sin y\,dx\,dy \rightarrow$ inner integral: $\int_0^2 e^x \sin y\,dx = e^x \sin y\Big|_0^2$
$e^2 \sin y - \sin y$

outer integral:

$\int_0^{\frac{\pi}{4}} e^x \sin y - \sin y\,dy = \int_0^{\frac{\pi}{4}} e^x \sin y\,dy - \int_0^{\frac{\pi}{4}} \sin y\,dy$

$= [e^x(-\cos y)]\Big|_0^{\frac{\pi}{4}} - [-\cos y]\Big|_0^{\frac{\pi}{4}}$

$= \left[-e^{\frac{\pi}{4}}\cos(\frac{\pi}{4}) - e^0\cos(0)\right] + \left[\cos(\frac{\pi}{4}) - \cos(0)\right]$

$= -e^{\frac{\pi}{4}}\cos(\frac{\pi}{4}) - 1 + \cos(\frac{\pi}{4}) - 1 = -\frac{e^{\frac{\pi}{4}}\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 2$

15.2 HW- #3, 5, 6, 7, 11, 19, 21, 25, 31, 33, 35, 37, 43, 49

3. $\int_0^1 \int_0^1 (1-x^2)\,dx\,dy \rightarrow \int_0^1 (1-x^2)\,dx = (x - \frac{x^3}{3})\Big|_0^1 = (1 - \frac{1}{3}) - 0$
$= \frac{2}{3}$

$\int_0^1 \frac{2}{3}\,dy = \frac{2}{3}y\Big|_0^1 = \frac{2}{3}$

5. $\int_0^2 \int_1^4 x^2 y\,dx\,dy \rightarrow \int_1^4 x^2 y^2\,dx = \frac{x^3 y^2}{3}\Big|_1^4 = \frac{4^3 y^2}{3} - \frac{y^2}{3} = \left[\frac{64y^2 - y^2}{3}\right.$

$\int_0^2 \frac{64}{3}y^2 - \int \frac{y^2}{3} = \frac{64}{3}\int_0^2 y^2 - \frac{1}{3}\int y^2 = \frac{64}{3}\left[\frac{y^3}{3}\right]\Big|_0^2 - \frac{1}{3}\left[\frac{y^3}{3}\right]\Big|_0^2$

$= \frac{64}{3}\left[\frac{8}{3}\right] - \frac{1}{3}\left[\frac{8}{3}\right] = \frac{512}{9} - \frac{8}{9} = 56$

6. same as #5

7. Same as #5.

11. $\iint_D \left(\frac{\sqrt{4-x^2}}{x} \right) dx\,dy$ $\quad \int_1^2 \frac{\sqrt{4-x^2}}{x}\,dx$ $\quad -\frac{1}{2}\int_1^2 \left(u^{1/2} \right) du$

$\int_0^{\sqrt{3}} \int_1^2 \left(\frac{\sqrt{4-x^2}}{x} \right) dx\,dy$ $\quad u = 4-x^2$ $\quad = -\frac{1}{2}\left[\frac{2}{3} u^{3/2} \right] \Big|_1^2$

$du = -2x\,dx$

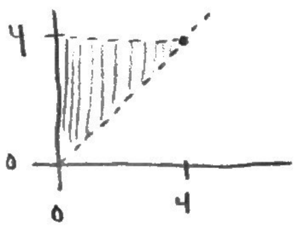
$= -\frac{1}{2}\left[\frac{2}{3}(2)^{3/2} - \frac{2}{3}(1)^{3/2} \right]$

19. $\int_1^{e^{x^2}} \int_0^1 x\,dx\,dy \rightarrow \int_0^1 x\,dx = \frac{x^2}{2} \Big|_0^1 = \frac{1}{2}$

$\int_1^{e^{x^2}} \frac{1}{2}\,dy = \frac{1}{2}y \Big|_1^{e^{x^2}} = \frac{1}{2}e^{x^2} - \frac{1}{2}(1) = \frac{e^{x^2}-1}{2}$

21. $\int_y^{y^2} (2xy)\,dy = xy^2 \Big|_y^{y^2} = xy^4 - xy^2$

25. $\int_0^4 \int_x^4 f(x,y)\,dy\,x$



31. $f(x,y) = (\ln y)^{-1} = \frac{1}{\ln(y)}$

$\int_{e^{\sqrt{x}}}^{e^x} \left(\frac{1}{\ln(y)} \right) dy = \left[\frac{1}{\ln(e^{\sqrt{x}})} - \frac{1}{\ln(e^x)} \right]$

~~u = ln y~~

~~du = 1/y dy~~

33. $\int_0^1 \int_y^1 \frac{\sin x}{x}\,dx\,dy$ $\quad \int_y^1 \frac{\sin x}{x}\,dx = \int_y^1 \sin x \cdot x^{-1}\,dx$

$= u = \sin x \quad dv = x^{-1}$

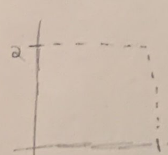
$du = \cos x \quad v = \ln x$

$= \sin x \ln x - \int \ln x \cos x$

35. $\int_0^1 \int_{y=x}^1 x e^{y^3}\,dy\,dx$

$\int_y^1 x e^{y^3}\,dy = \frac{x e^{y^3}}{3y^2}\Big|_{y=x}^{1} = \left[\frac{xe}{3} - \frac{x e^{x^3}}{3x^2}\right]$

37. $\iint_D e^{x+y}\,dA$ $\quad \int_0^2 \int_0^2 e^{x+y}\,dx\,dy$



$[e^{x+y}]\,|_0^2 = e^{2+y} - e^y$

$\int_0^2 [e^{2+y} - e^y]\,dy = [e^{2+y} - e^y]\,|_0^2$

$e^4 - e^2 - e^2 - 1 = e^4 - 2e^2 - 1$

43. $\int_{\frac{x}{2}}^{y} \int_1^2 \frac{\sin y}{y}\,dx\,dy$ $\quad \left[\frac{\sin y}{y}\right][x]\,|_1^2 = \frac{2\sin y}{y} - \frac{\sin y}{y} = \frac{\sin y}{y}$

49. $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} [(8 - x^2 - y^2) - (x^2 + y^n)\,dy)\,dx$