15.1, 15.2 HW

10/20/20

15.1: # 9, 15, 21, 23, 25, 31, 33, 35, 37, 41

9. $\iint_R (15-3x)\, dA$; $R = [0 \times 5] \times [0,3]$

$A = \frac{1}{2} \cdot 5 \cdot 15 = 75/2$

$V = \ell A = 3\,(75/2) = 225/2$

15. $\iint_R x^3\, dA$; $R = [-4,4] \times [0,5]$

$f(-x,y) = (-x)^3 = -x^3 = -f(x,y)$

$\iint_R x^3\, dA = 0$

21. $$\int_1^3 \int_0^2 x^3 y\, dy\, dx = \int_1^3 x^3 \frac{y^2}{2}\Big|_{y=0}^{2} dx = \int_1^3 x^3 \left(\frac{2^2}{2} - 0\right) dx$$

$$= \int_1^3 2x^3\, dx = \frac{x^4}{2}\Big|_1^3 = 40$$

23. $$\int_{-1}^1 \int_0^{\pi} x^2 \sin y\, dy\, dx = \int_{-1}^1 x^2(-\cos y)\Big|_{y=0}^{\pi} dx = \int_{-1}^1 x^2(-\cos \pi + \cos 0)\, dx$$

$$= \int_{-1}^1 x^2(1+1)\, dx = \int_{-1}^1 2x^2 dx = \frac{2}{3}x^3\Big|_{-1}^1 = \frac{2}{3}\left(1^3 - (-1)^3\right) = \frac{4}{3}$$

25. $$\int_2^6 \int_1^4 x^2 dx\, dy = \int_2^6 \int_1^4 x^2 \cdot 1\, dx\, dy = \left(\int_1^4 x^2 dx\right)\left(\int_2^6 1\, dy\right)$$

$$= \left(\frac{x^3}{3}\Big|_1^4\right)\left(y\Big|_2^6\right) = \left(\frac{4^3}{3} - \frac{1^3}{3}\right)(6-2) = 21 \cdot 4 = 84$$

31. $$\int_1^2 \int_0^4 \frac{dy\, dx}{x+y} = \int_1^2 \left(\int_0^4 \frac{dy}{x+y}\right) dx = \int_1^2 \ln(x+y)\Big|_{y=0}^{4} dx$$

$$= \int_1^2 (\ln(x+4) - \ln x)\, dx \quad \left(\text{Use Integral Formula: } \int \ln(x+a)\, dx = (x+a)(\ln(x+a)-1)+C\right)$$

$$\int_1^2 \int_0^4 \frac{dy\, dx}{x+y} = (x+4)(\ln(x+4) - 1) - x(\ln x - 1)\Big|_1^2 = 6(\ln 6 - 1) -$$

$$2(\ln 2 - 1) - (5(\ln 5 - 1) - (\ln 1 - 1)) = 6\ln 6 - 2\ln 2 - 5\ln 5 \approx 1.31$$

33. $$\int_0^4 \int_0^5 \frac{dy\, dx}{\sqrt{x+y}} = \int_0^4 \left(\int_0^5 \frac{dy}{\sqrt{x+y}}\right) dx = \int_0^4 \left(2\sqrt{x+y}\Big|_{y=0}^{5}\right) dx$$

$$= 2\int_0^4 \left(\sqrt{x+5} - \sqrt{x}\right) dx = 2\left(\frac{2}{3}(x+5)^{3/2} - \frac{2}{3}x^{3/2}\right)\Big|_0^4$$

$$= 2\left(\frac{2}{3} \cdot 27 - \frac{2}{3} \cdot 8\right) - 2\left(\frac{2}{3} \cdot 5^{3/2} - 0\right) = 36 - \frac{32}{3} - \frac{20}{3}\sqrt{5} \approx 10.426$$

35. $\int_1^2\int_1^3 \frac{\ln(xy)\,dy\,dx}{y} = \int_1^2 \left(\frac{1}{2}[\ln(xy)]^2 \Big|_1^3 \right) dx$

$= \frac{1}{2}\int_1^2 [\ln(3x)]^2 - [\ln(x)]^2\,dx = \frac{1}{2}\int_1^2 [\ln(3x)]^2 dx - \frac{1}{2}\int_1^2 [\ln(x)]^2\,dx$

$= \frac{1}{2}\left[x(\ln 3x)^2 \Big|_1^2 - 2\int_1^2 \ln(3x)\,dx \right] - \frac{1}{2}\left[x(\ln x)^2 \Big|_1^2 - 2\int_1^2 \ln x\,dx \right]$

$= \frac{1}{2}\left[2(\ln 6)^2 - (\ln 3)^2\right] - \int_1^2 \ln(3x)dx - \frac{1}{2}\left[2(\ln 2)^2 - 0\right] + \int_1^2 \ln x\,dx$

$= (\ln 6)^2 - \frac{1}{2}(\ln 3)^2 - \left[x\ln(3x) - x \Big|_1^2 \right] - (\ln 2)^2 + \left[x\ln x - x \Big|_1^2 \right]$

$= (\ln 6)^2 - 1/2\,(\ln 3)^2 - (\ln 2)^2 - (2\ln 6 - 2 - \ln 3 + 1) + (2\ln 2 - 2 - 0 + 1)$

$= (\ln 6)^2 - 1/2\,(\ln 3)^2 - (\ln 2)^2 - 2\ln 6 + \ln 3 + 1 + 2\ln 2 - 2 + 1$

$= (\ln 6)^2 - 1/2\,(\ln 3)^2 - (\ln 2)^2 - 2\ln 6 + \ln 3 + 2\ln 2 \approx 1.028$

37. $\iint_R \frac{x}{y}\,dA \;;\; R = [-2,4]\times[1,3]$

$\int_{-2}^4\int_1^3 \frac{x}{y}\,dy\,dx = \int_{-2}^4 x\,dx \cdot \int_1^3 \frac{1}{y}\,dy = \left(\frac{1}{2}x^2 \Big|_{-2}^4\right)\left(\ln y \Big|_1^3\right)$

$= 1/2\,(16-4)\cdot(\ln 3 - \ln 1) = 6\ln 3$

41. $\iint_R e^x \sin y\,dA \;;\; R = [0,2]\times[0, \pi/4]$

$\int_0^{\pi/4}\int_0^2 e^x \sin y\,dx\,dy = \left(\int_0^2 e^x dx\right)\left(\int_0^{\pi/4} \sin y\,dy\right) = \left(e^x \Big|_0^2\right)\left(-\cos y \Big|_0^{\pi/4}\right)$

$= (e^2 - e^0)(-\cos \pi/4 + \cos 0) = (e^2 - 1)(1 - \sqrt{2}/2) \approx 1.87$

15.2: # 3, 5, 7, 11, 19, 25, 31, 33, 35, 37, 43, 49

3. $D:\; 0 \le x \le 1,\; 0 \le y \le 1 - x^2$

$y = 1 - x^2 \Rightarrow x^2 = 1 - y \Rightarrow x = \sqrt{1-y}$

$D:\; 0 \le y \le 1,\; 0 \le x \le \sqrt{1-y}$

(i) $\iint_D xy\,dA = \int_0^1\int_0^{1-x^2} xy\,dy\,dx = \int_0^1 \frac{xy^2}{2}\Big|_{y=0}^{1-x^2} dx = \int_0^1 \frac{x}{2}\left((1-x^2)^2 - 0^2\right)dx$

$= \int_0^1 \frac{x(1-x^2)^2}{2}\,dx = \frac{1}{2}\int_0^1 (x - 2x^3 + x^5)\,dx = \frac{1}{2}\left(\frac{x^2}{2} - \frac{x^4}{2} + \frac{x^6}{6}\right)\Big|_0^1 = \frac{1}{12}$

(2) $\iint_D xy\,dA = \int_0^1 \int_0^{\sqrt{1-y}} xy\,dx\,dy = \int_0^1 \frac{yx^2}{2}\Big|_{x=0}^{\sqrt{1-y}} dy = \int_0^1 \frac{y}{2}\left(\left(\sqrt{1-y}\right)^2 - 0^2\right) dy$

$= \int_0^1 \frac{y}{2}(1-y)\,dy = \int_0^1 \left(\frac{y}{2} - \frac{y^2}{2}\right) dy = \frac{y^2}{4} - \frac{y^3}{6}\Big|_0^1 = \frac{1}{4} - \frac{1}{6} = \frac{1}{12}$ ((1) = (2))

5. $y - 0 = \frac{2-0}{0-4}(x-4) \Rightarrow y = -\frac{1}{2}x + 2$ $\left(D:\ 0 \le x \le 4,\ -\frac{1}{2}x + 2 \le y \le 2\right)$

$\iint_D x^2 y\,dA = \int_0^4 \int_{-\frac{x}{2}+2}^{2} x^2 y\,dy\,dx = \int_0^4 \frac{x^2y^2}{2}\Big|_{y=-\frac{x}{2}+2}^{2} dx$

$= \int_0^4 \frac{x^2}{2}\left(2^2 - \left(-\frac{x}{2}+2\right)^2\right) dx = \int_0^4 \left(x^3 - \frac{x^4}{8}\right) dx = \frac{x^4}{4} - \frac{x^5}{40}\Big|_0^4$

$= 4^4/4 - 4^5/40 = 192/5 = 38.4$

7. $D:\ 0 \le y \le 2,\ y \le x \le 4$

$\iint_D x^2 y\,dA = \int_0^2 \int_y^4 x^2 y\,dx\,dy = \int_0^2 \frac{x^3 y}{3}\Big|_{x=y}^{x=4} dy = \int_0^2 \frac{y}{3}(4^3 - y^3)\,dy$

$= \int_0^2 \left(\frac{64y}{3} - \frac{y^4}{3}\right) dy = \frac{32}{3}y^2 - \frac{y^5}{15}\Big|_0^2 = \frac{32 \cdot 2^2}{3} - \frac{2^5}{15} = \frac{608}{15} \approx 40.53$

11. $D:\ 1 \le x \le 2,\ 0 \le y \le \sqrt{4-x^2}$

$\iint_D \frac{y}{x}\,dA = \int_1^2 \int_0^{\sqrt{4-x^2}} \frac{y}{x}\,dy\,dx = \int_1^2 \frac{y^2}{2x}\Big|_0^{\sqrt{4-x^2}} dx = \int_1^2 \left(\frac{4-x^2}{2x} - 0\right) dx$

$= \int_1^2 \left(\frac{2}{x} - \frac{1}{2}x\right) dx = \left(2\ln|x| - \frac{1}{4}x^2\right)\Big|_1^2 = 2\ln 2 - \frac{1}{4} \cdot 2^2 - 0 + \frac{1}{4} \cdot 1^2$

$= 2\ln 2 - 3/4$

19. $f(x,y) = x$; $0 \le x \le 1,\ 1 \le y \le e^{x^2}$

$\iint_D x\,dA = \int_0^1 \int_1^{e^{x^2}} x\,dy\,dx = \int_0^1 xy\Big|_{y=1}^{e^{x^2}} dx = \int_0^1 \left(xe^{x^2} - x \cdot 1\right) dx$

$= \int_0^1 xe^{x^2}dx - \int_0^1 x\,dx = \int_0^1 xe^{x^2}dx - \frac{x^2}{2}\Big|_0^1 = \int_0^1 xe^{x^2}dx - \frac{1}{2}$ $u = x^2$

$= \iint_D x\,dA = \frac{e-1}{2} - \frac{1}{2} = \frac{e-2}{2} \approx 0.359$

★ $\int_0^4 \int_x^4 f(x,y)\,dy\,dx = \int_0^4 \int_0^y f(x,y)\,dx\,dy$

31. $y = e^x \Rightarrow x = \ln y$ $\quad\left(D: 1 \le y \le 2,\ \ln^2 y \le x \le \ln y\right)$

$y = e^{\sqrt{x}} \Rightarrow \sqrt{x} = \ln y \Rightarrow x = \ln^2 y$

$$\iint_D (\ln y)^{-1}\, dA = \int_1^2 \int_{\ln^2 y}^{\ln y} (\ln y)^{-1}\, dx\, dy = \int_1^2 (\ln y)^{-1} x \Big|_{x=\ln^2 y}^{\ln y} dy$$

$$= \int_1^2 (\ln y)^{-1}(\ln y - \ln^2 y)\, dy = \int_1^2 (1 - \ln y)\, dy = \int_1^2 1\, dy - \int_1^2 \ln y\, dy$$

$$= y\Big|_1^2 - y(\ln y - 1)\Big|_1^2 = 1 - (2(\ln 2 - 1) - 1(\ln 1 - 1)) = 2 - 2\ln 2 \approx 0.614$$

33. $\displaystyle\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$ $\quad\left(D: 0 \le x \le 1,\ 0 \le y \le x\right)$

$$\int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx = \int_0^1 \frac{\sin x}{x} y \Big|_{y=0}^{x} dx = \int_0^1 \frac{\sin x}{x}(x - 0)\, dx$$

$$= \int_0^1 \sin x\, dx = -\cos x \Big|_0^1 = 1 - \cos 1 \approx 0.46$$

35. $\displaystyle\int_0^1 \int_{y=x}^1 x e^{y^3}\, dy\, dx$ $\quad\left(D: 0 \le y \le 1,\ 0 \le x \le y\right)$

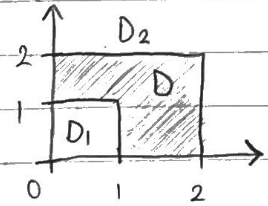
$$\int_0^1 \int_0^y x e^{y^3}\, dx\, dy = \int_0^1 \frac{x^2}{2} e^{y^3} \Big|_{x=0}^{y} dy = \int_0^1 e^{y^3}\left(\frac{y^2}{2} - 0\right) dy = \int_0^1 \frac{1}{2} e^{y^3} y^2\, dy$$

$u = y^3$
$du = 3y^2 dy$

$$\Rightarrow \int_0^1 \int_0^y x e^{y^3}\, dx\, dy = \int_0^1 \frac{1}{2} e^{y^3} y^2\, dy = \int_0^1 e^u \cdot \frac{1}{6}\, du = \frac{e^u}{6}\Big|_0^1$$

$= (e - 1)/6 \approx 0.286$

37.



$$\iint_{D_2} e^{x+y}\, dA = \iint_{D_1} e^{x+y}\, dA + \iint_D e^{x+y}\, dA$$

$$\iint_D e^{x+y}\, dA = \iint_{D_2} e^{x+y}\, dA - \iint_{D_1} e^{x+y}\, dA = \int_0^2 \int_0^2 e^{x+y}\, dx\, dy$$

$$- \int_0^1 \int_0^1 e^{x+y}\, dx\, dy = \int_0^2 e^{x+y} \Big|_{x=0}^{2} dy - \int_0^1 e^{x+y} \Big|_{x=0}^{1} dy$$

$$= \int_0^2 (e^{2+y} - e^y)\, dy - \int_0^1 (e^{1+y} - e^y)\, dy = e^{2+y} - e^y \Big|_{y=0}^{2} - (e^{1+y} - e^y)\Big|_{y=0}^{1}$$

$$= e^4 - e^2 - (e^2 - e^0) - (e^2 - e - (e - e^0)) = e^4 - 3e^2 + 2e \approx 37.87$$

43. $D: 1 \le y \le 2 \quad , \quad y \le x \le 2y$

$$\iint_D \frac{\sin y}{y}\, dA = \int_1^2 \int_y^{2y} \frac{\sin y}{y}\, dx\, dy = \int_1^2 \frac{\sin y}{y} x \Big|_{x=y}^{2y} dy = \int_1^2 \frac{\sin y}{y}(2y-y)\, dy$$

$$= \int_1^2 \frac{\sin y}{y} \cdot y\, dy = \int_1^2 \sin y\, dy = -\cos y \Big|_1^2 = \cos 1 - \cos 2 \approx 0.956$$

49. $$\iint_D ((8-x^2-y^2)-(x^2+y^2))\, dA = \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (8-2x^2-2y^2)\, dy\, dx$$