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Exercise 14.8

Q5. $f(x,y) = x^2+y^2$

$2x+3y=6$

Answer: $f_x=2x$ $\quad f_y=2y$ $\quad \nabla f = (2x,2y)$

$g_x=2$ $\quad g_y=3$ $\quad \nabla g = (2,3)$

$\nabla f = \lambda \nabla g$

$(2x,2y) = \lambda(2,3)$

$2x=2\lambda$ $\quad 2y=3\lambda$

$x=\lambda$ $\quad y=\frac{3}{2}\lambda$

$\because 2x+3y=6$

plug-in $= 2(\lambda)+3(\frac{3}{2}\lambda)=6$

$\frac{13}{2}\lambda=6$

$\lambda=\frac{12}{13}$

$\therefore x=\frac{12}{13}$ $\quad y=\frac{18}{13}$

the point is $(\frac{12}{13}, \frac{18}{13})$

$f(\frac{12}{13}, \frac{18}{13}) = (\frac{12}{13})^2+(\frac{18}{13})^2 = \frac{468}{169}$

$= \frac{36}{13}$

therefore the minimum number is $\frac{36}{13}$.

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Q7. $f(x,y)=xy$, $4x^2+9y^2=32$

$f_x=y$ $f_y=x$ $\nabla f=(y,x)$

$g_x=8x$ $g_y=18y$ $\nabla g=(8x,18y)$

$\nabla f=\nabla g\cdot\lambda$

$(y,x)=(8x,18y)\lambda$

$y=8x\cdot\lambda$ $x=18y\cdot\lambda$

$x\cdot y=8x\cdot 18y\cdot\lambda^2$

$1=144\lambda^2$

$\lambda^2=\frac{1}{144}$

$\lambda=\pm\frac{1}{12}$

$y=8x\cdot\frac{1}{12}=\frac{2}{3}x$ or $8x\cdot(-\frac{1}{12})=-\frac{2x}{3}$

$x=18y\cdot\frac{1}{12}=\frac{3}{2}y$ or $-\frac{3}{2}y$

$(\frac{3}{2}y,\frac{2}{3}x)$ & $(-\frac{3}{2}y,-\frac{2}{3}x)$

$4(\frac{3}{2}y)^2+9(\frac{2}{3}x)^2=32$

$9y^2+4x^2=32$

$9y^2+9y^2=32$

$y^2=\frac{32}{18}$

$y=\sqrt{\frac{16}{9}}=\frac{4}{3}$ or $-\frac{4}{3}$

$4x^2+4x^2=32$

$x^2=\cancel{8}\,4$

$x=\cancel{\pm 2\sqrt{2}}$ ± 2

$\therefore$ the point is $(\cancel{2\sqrt{2}}\,2,\frac{4}{3})$ or $(-\cancel{2\sqrt{2}}\,2,-\frac{4}{3})$

$(2,-\frac{4}{3})$, $(-2,\frac{4}{3})$

$f(x,y)=xy=\cancel{2\sqrt{2}\cdot\frac{4}{3}=\frac{8\sqrt{2}}{3}}$ $2\cdot\frac{4}{3}=\frac{8}{3}$ or $(-2)\cdot\frac{8}{3}=-\frac{8}{3}$

$\therefore$ the maximum is $\frac{8}{3}$, minimum is $-\frac{8}{3}$

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Q9. $f(x,y) = x^2+y^2$ $\quad x^4+y^4=1$

Answer: $f_x = 2x \quad f_y = 2y \quad \nabla f = (2x, 2y)$

$g_x = 4x^3 \quad g_y = 4y^3 \quad \nabla g = (4x^3, 4y^3)$

$\nabla f = \nabla g \cdot \lambda$

$(2x, 2y) = (4x^3, 4y^3)\lambda$

$2x = 4x^3\lambda \qquad 2y = 4y^3\lambda$

$1 = 2x^2\lambda \qquad 1 = 2y^2\lambda$

$x^2 = \frac{1}{2\lambda} \qquad y^2 = \frac{1}{2\lambda}$

$x = \pm\sqrt{\frac{1}{2\lambda}} \qquad y = \pm\sqrt{\frac{1}{2\lambda}}$

$x^4+y^4=1$

$\left(\sqrt{\frac{1}{2\lambda}}\right)^4 + \left(\sqrt{\frac{1}{2\lambda}}\right)^4 = 1$

$\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 1$

$\frac{1}{2\lambda^2} = 1$

$1 = 2\lambda^2$

$\lambda^2 = \frac{1}{2}$

$\lambda = \pm\sqrt{\frac{1}{2}} = \frac{\sqrt{2}}{2}$ or $-\frac{\sqrt{2}}{2}$

($-\frac{1}{2}$ DNE)

$x = \sqrt{\frac{1}{\sqrt{2}}} \qquad y = \sqrt{\frac{1}{\sqrt{2}}}$

$= \sqrt{\frac{\sqrt{2}}{2}} \qquad = \sqrt{\frac{\sqrt{2}}{2}}$

or $\sqrt{\frac{1}{-\sqrt{2}}} \qquad = \sqrt{\frac{1}{-\sqrt{2}}}$

$\therefore f(x,y) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

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Q11. $f(x,y,z) = 3x+2y+4z \qquad x^2+2y^2+6z^2=1$

$f_x = 3 \quad f_y = 2 \quad f_z = 4 \qquad \nabla f = (3,2,4)$

$g_x = 2x \quad g_y = 4y \quad g_z = 12z \quad \nabla g = (2x, 4y, 12z)$

$\nabla f = \nabla g \cdot \lambda$

$(3,2,4) = (2x, 4y, 12z)\lambda$

$3 = 2x\lambda \qquad 2 = 4y\lambda \qquad 4 = 12z\lambda$

$x = \frac{3}{2\lambda} \qquad y = \frac{1}{2\lambda} \qquad z = \frac{1}{3\lambda}$

$$\left(\frac{3}{2\lambda}\right)^2 + 2\left(\frac{1}{2\lambda}\right)^2 + 6\left(\frac{1}{3\lambda}\right)^2 = 1$$

$$\frac{9}{4\lambda^2} + \frac{2}{4\lambda^2} + \frac{6}{9\lambda^2} = 1$$

$$\frac{11}{4\lambda^2} + \frac{6}{9\lambda^2} = 1$$

$$\frac{99+24}{36\lambda^2} = 1$$

$$123 = 36\lambda^2$$

$$\lambda^2 = \frac{41}{12}$$

$$\lambda = \pm\sqrt{\frac{41}{12}} = \pm\frac{\sqrt{123}}{6}$$

$x = \frac{3\sqrt{123}}{41} \qquad y = \frac{\sqrt{123}}{41} \qquad z = \frac{2}{\sqrt{123}}$

(positive) $f(x,y,z) = 3\left(\frac{3\sqrt{123}}{41}\right) + 2\left(\frac{\sqrt{123}}{41}\right) + 4\left(\frac{2}{\sqrt{123}}\right) = 3.696$

(neg). $f(x,y,z) = -3.696$

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Q13. $f(x,y,z) = xy + 2z, \quad x^2+y^2+z^2 = 36$

$f_x = y \quad f_y = x \quad f_z = 2 \quad \nabla f = (y, x, 2)$

$g_x = 2x \quad g_y = 2y \quad g_z = 2z \quad \nabla g = (2x, 2y, 2z)$

$(y, x, 2) = \lambda(2x, 2y, 2z)$

$2 = 2z\lambda$

$\left\{ z = \frac{1}{\lambda} \right.$

$\cdot \lambda = \frac{1}{z}$

$2xy = 8xyz\lambda^3$

$1 = 4z\lambda^3$

$z = \frac{1}{4\lambda^3}$

$\lambda^3 = \frac{1}{4z}$

$\lambda = \sqrt[3]{\frac{1}{4z}}$

$y = 2x\lambda$

$\begin{cases} y = \frac{2x}{z} \\ x = \frac{2y}{z} \end{cases}$

$xyz = \frac{2y}{z} \cdot \frac{2x}{z} \cdot z = \frac{4xy}{z}$

$z = \frac{4}{z}$

$\lambda = \frac{1}{\pm 2}$

$z^2 = 4$

$z = \pm 2$

$\because x = y \quad \therefore x = y = \pm 4$

$\therefore f(x,y,z) = (\pm 4, \pm 4, 2) = 20$ Max

$f(x,y,z) = (\pm 4, \pm 4, -2) = -20$ Mini

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Q15. $f(x,y,z) = xy + xz \qquad x^2+y^2+z^2=4$

$f_x = y+z \quad f_y = x \quad f_z = x \qquad (y+z, x, x)$

$g_x = 2x \quad g_y = 2y \quad g_z = 2z \qquad (2x, 2y, 2z)$

$(y+z, x, x) = (2x, 2y, 2z)\lambda$

$y+z = 2x\lambda \qquad \frac{x}{2\lambda}\cdot 2 = 2x\lambda$

$x = 2y\lambda \quad y = \frac{x}{2\lambda} \qquad \frac{x}{\lambda} = 2x\lambda$

$x = 2z\lambda \quad z = \frac{x}{2\lambda} \qquad x = 2x\lambda^2$

$1 = 2\lambda^2$

$\lambda^2 = \frac{1}{2}$

$\lambda = \pm\sqrt{\frac{1}{2}} = \pm\frac{\sqrt{2}}{2}$

$\therefore \cancel{x=}\quad 2y = 2(2y)\lambda^2$

$y = 2y\cdot\left(\frac{\sqrt{2}}{2}\right)^2$

$y = \cancel{\sqrt{2}y}\ y$

$x = 2y\left(\pm\frac{\sqrt{2}}{2}\right) = \pm\sqrt{2}y$

$\therefore (\pm\sqrt{2}y)^2 + y^2 + y^2 = 4$

$y = 1$

$x = \pm\sqrt{2} \quad z = 1$

$\therefore f(x,y,z)$ should be $(\sqrt{2},1,1) = 2\sqrt{2}$ Max.

or $(-\sqrt{2},1,1) = -2\sqrt{2}$ Mini