Ch. 14.8 HW #5, 7, 9, 11, 13, 15 due 10/18

5.) $f(x,y) = x^2 + y^2$, $g(x,y) = 2x + 3y = 6$

$\nabla f(x,y) = \langle 2x, 2y \rangle$ $g(x,y) = \langle 2, 3 \rangle$

$\nabla f(x,y) = \lambda \nabla g(x,y) \rightarrow \langle 2x, 2y \rangle = \lambda \langle 2, 3 \rangle$

$2x = 2\lambda$ $2y = \lambda 3 \rightarrow 2y = 3x$

$x = \lambda$ $y = 3x/2$

$2x + 3(3x/2) = 6 \rightarrow \frac{2}{13} \cdot \frac{13}{2} x = 6 \cdot \frac{2}{13}$ $x = 12/13$

$\frac{4}{2}x + \frac{9}{2}x = 6$

$2(\frac{12}{13}) + 3y = \frac{78}{13}$

$\frac{1}{3} \cdot 3y = \cancel{54}^{18}/13 \cdot \frac{1}{3}$,

$y = 18/13$

The Extreme value at $(12/13, 18/13)$ is $f(12/13, 18/13) = (\frac{12}{13})^2 + (\frac{18}{13})^2$

$= \frac{12^2 + 18^2}{13^2} = 468/169 = 2.77$

7.) $f(x,y) = xy$, $g(x,y) = 4x^2 + 9y^2 = 32$

$\nabla f(x,y) = \langle y, x \rangle$, $\nabla g(x,y) = \langle 8x, 18y \rangle$

$\langle y, x \rangle = \lambda \langle 8x, 18y \rangle$

$y = \lambda 8x$ $x = \lambda 18y$ $x^2 = 9y^2/4$

$\lambda = y/8x$ $x = \frac{9y^2}{4x}$

$4(\frac{9y^2}{4}) + 9y^2 = 32$

$18y^2 = 32 \rightarrow y^2 = \sqrt{16/9}$

$y = -4/3, 4/3$

$(-2, -4/3), (2, -4/3), (-2, 4/3), (2, 4/3)$

$4x^2 + 9(\frac{16}{9}) = 32$

$4x^2 = 16$ $x^2 = 4$ $x = -2, 2$

$f(-2, -4/3) = 8/3 = f(2, 4/3)$

$f(-2, 4/3) = f(2, -4/3) = -8/3$

The max is $8/3$ at $(-2, -4/3)$ and $(2, 4/3)$. The min is $-8/3$ at $(-2, 4/3)$ and $(2, -4/3)$

9.) $f(x,y) = x^2 + y^2$, $x^4 + y^4 = 1$

$\nabla f(x,y) = \langle 2x, 2y \rangle$, $\nabla g(x,y) = \langle 4x^3, 4y^3 \rangle$

$\langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle$

$2x = \lambda 4x^3$ $2y = \lambda 4y^3 \rightarrow 4yx^2 = 4y^3$

$\lambda = \frac{1}{2x^2}$ $2y = 4y^3/2x^2$ $x^2 = y^2$

$y^4 + y^4 = 1$

$2y^4 = 1$

$y^4 = \frac{1}{2}$

$y = -2^{-1/4}, 2^{-1/4}$

$x^4 + \frac{1}{2} = 1$

$x^4 = \frac{1}{2} \rightarrow x = -2^{-1/4}, 2^{1/4}$

$f(-2^{-1/4}, -2^{-1/4}) = 1/\sqrt{2} + 1/\sqrt{2} = 2/\sqrt{2} \cdot \sqrt{2}/\sqrt{2} = 2\sqrt{2}/2 = \sqrt{2}$

$f(-2^{-1/4}, 2^{-1/4}) = \sqrt{2} = f(2^{-1/4}, -2^{-1/4}) = f(2^{-1/4}, 2^{-1/4})$

The max is $\sqrt{2}$

? 11.) $f(x,y,z) = 3x + 2y + 4z$, $g(x,y) = x^2 + 2y^2 + 6z^2 = 1$

$\nabla f(x,y,z) = \langle 3,2,4 \rangle$, $\nabla g(x,y) = \langle 2x, 4y, 12z \rangle$

$\langle 3,2,4 \rangle = \lambda \langle 2x, 4y, 12z \rangle$

$3 = \lambda 2x$ $\quad 2 = \lambda 4y$ $\quad 4 = \lambda 12z \rightarrow x = 3/2\lambda, y = 1/2\lambda, z = \frac{1}{3\lambda}$

$24 = \lambda^3 96 xyz \rightarrow xyz = 1/4\lambda^3$???

13.) $f(x,y,z) = xy + 2z$, $g(x,y,z) = x^2 + y^2 + z^2 = 36$

$\nabla f(x,y,z) = \langle y, x, 2 \rangle$ $\quad \nabla g(x,y,z) = \langle 2x, 2y, 2z \rangle$

$\langle y, x, 2 \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$y = \lambda 2x$ $\quad x = \lambda 2y$ $\quad 2 = \lambda 2z$ $\quad x^2 + x^2 + 4 = 36$

$xy = 4\lambda^2 xy$ $\quad y = x$ $\quad z = 2$ $\quad 2x^2 = 32$ $\quad x^2 = 16$ $\quad x = \pm 4$

$\lambda^2 = 1/4 = 1/2$ $\quad (4,4,2), (4,-4,2), (-4,4,2), (-4,-4,2)$

$f(4,4,2) = 16 + 4 = 20 = f(-4,-4,2)$

$f(4,-4,2) = f(-4,4,2) = -16 + 4 = -12$

max is 20 and min is -12

15.) $f(x,y,z) = xy + xz$, $g(x,y,z) = x^2 + y^2 + z^2 = 4$

$\nabla f(x,y,z) = \langle y+z, x, x \rangle$, $\nabla g(x,y,z) = \langle 2x, 2y, 2z \rangle$

$\langle y+z, x, x \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$y + z = \lambda 2x$, $\quad x = \lambda 2y$, $\quad x = \lambda 2z$ $\quad (\sqrt{2}\, z)^2 + 2z^2 = 4$

$2\lambda y = 2\lambda z$ $\quad 2z = 2\lambda(2\lambda z)$ $\quad 2z^2 + 2z^2 = 4 \rightarrow z^2 = 1 \rightarrow z = 1$

$y = z$ $\quad 2z = 4\lambda^2 z$ $\quad (\sqrt{2}, 1, 1)$

$x = \sqrt{2}\, z$ $\quad \lambda^2 = 1/2 = \frac{\sqrt{2}}{2}$

$f(\sqrt{2}, 1, 1) = \sqrt{2} \cdot 1 + \sqrt{2} \cdot 1 = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$ max