5. 14.8 $f_x=2x$ $f_y=2y$.

$\nabla f=\langle 2x,2y\rangle$

$g_x=2$ $g_y=3$.

$\nabla g=\langle 2,3\rangle$

$\nabla f=L\nabla g$

$\langle 2x,2y\rangle=L\langle 2,3\rangle$

$2x=2L$ $2y=3L$ $2x+3y=6$

$x=L$ $y=\frac{3L}{2}$

$2L+3\cdot\frac{3L}{2}=6$

$2L+\frac{9L}{2}=6$

$L=\frac{12}{13}$

$x=\frac{12}{13}$ $y=\frac{18}{13}$

$f(x,y)=x^2+y^2=(\frac{12}{13})^2+(\frac{18}{13})^2=\frac{36}{13}$

the minimum vale is $\frac{36}{13}$, no max

9. $f_x=2x$ $f_y=2y$.

$\nabla f=\langle 2x,2y\rangle$.

$g_x=4x^3$ $g_y=\cancel{4x}4y^3$.

$\nabla g=\langle 4x^3,4y^3\rangle$

$\nabla f=L\nabla g$

$\langle 2x,2y\rangle=L\langle 4x^3,4y^3\rangle$

$2x=4Lx^3$ $2y=4Ly^3$ $x^4+y^4=1$

$1=2Lx^2$ $1=2Ly^2$

$x^2=\frac{1}{2L}$ $y^2=\frac{1}{2L}$ $y=\pm\sqrt{\frac{1}{2L}}$

$x=\pm\sqrt{\frac{1}{2L}}$

$(\frac{1}{2L})^2+(\frac{1}{2L})^2=1$ $\frac{1}{4L^2}+\frac{1}{4L^2}=1$

7. $f_x=y$ $f_y=x$

$\nabla f=\langle y,x\rangle$

$g_x=8x$ $g_y=18y$.

$\nabla g=\langle 8x,18y\rangle$

$\nabla f=L\nabla g$

$\langle y,x\rangle=L\langle 8x,18y\rangle$

$y=8Lx$ $x=18Ly$. $4x^2+9y^2=32$

$y=144L^2y$

$1=144L^2$

$L=\frac{1}{12}$

$y=8\times\frac{1}{12}x$ $x=18\times\frac{1}{12}y$

$=\frac{2}{3}x$ $=\frac{3}{2}y$.

$4x^2+9\cdot\frac{4}{9}x^2=32$

$x=\pm2$ $y=\pm\frac{4}{3}$

$2\times\frac{4}{3}=\frac{8}{3}$ $-2\times(-\frac{4}{3})=\frac{8}{3}$

$-2\times\frac{4}{3}=-\frac{8}{3}$ $2\times(-\frac{4}{3})=-\frac{8}{3}$

maximum is $\frac{8}{3}$ minimum is $-\frac{8}{3}$

$\frac{2}{4L^2}=1$

$2=4L^2$

$\frac{1}{2}=L^2$

$L=\pm\frac{1}{\sqrt{2}}$

$x=\pm\sqrt{\frac{1}{2\times\frac{1}{\sqrt{2}}}}$

$y=\pm\sqrt{\frac{1}{2\times\frac{1}{\sqrt{2}}}}$

11. $f_x = 3 \quad f_y = 2 \quad f_z = 4$

$\nabla f = \langle 3, 2, 4 \rangle$

$g_x = 2x \quad g_y = 4y \quad g_z = 12z$

$\nabla g = \langle 2x, 4y, 12z \rangle$

$\nabla f = L \nabla g$

$\langle 3, 2, 4 \rangle = L \langle 2x, 4y, 12z \rangle$

$3 = 2Lx \quad 2 = 4Ly \quad 4 = 12Lz$

$x^2 + 2y^2 + 6z^2 = 1$

$x = \frac{3}{2L} \quad y = \frac{2}{4L} = \frac{1}{2L} \quad z = \frac{4}{12L} = \frac{1}{3L}$

$\left(\frac{3}{2L}\right)^2 + 2 \cdot \left(\frac{1}{2L}\right)^2 + 6 \cdot \left(\frac{1}{3L}\right)^2 = 1$

$L = \pm \frac{\sqrt{123}}{6}$

$x = \pm \frac{9}{\sqrt{123}} \quad y = \pm \frac{3}{\sqrt{123}} \quad z = \pm \frac{2}{\sqrt{123}}$

$3 \cdot \frac{9}{\sqrt{123}} + 2 \cdot \frac{3}{\sqrt{123}} + 4 \cdot \frac{2}{\sqrt{123}} \approx 3.7$

$3 \cdot \left(-\frac{9}{\sqrt{123}}\right) + 2 \cdot \left(-\frac{3}{\sqrt{123}}\right) + 4\left(-\frac{2}{\sqrt{123}}\right) \approx -3.7$

maximum 3.7 minimum −3.7

13. $f_x = y \quad f_y = x \quad f_z = 2$

$\nabla f = \langle y, x, 2 \rangle$

$g_x = 2x \quad g_y = 2y \quad g_z = 2z$

$\nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = L \nabla g$

$\langle y, x, 2 \rangle = L \langle 2x, 2y, 2z \rangle$

$y = 2Lx \quad x = 2Ly \quad 2 = 2zL$

$x^2 + y^2 + z^2 = 36$

$x = 2L \cdot 2Lx$

$\cancel{x} = 4L^2 \cancel{x}$

$1 = 4L^2$

$L^2 = \frac{1}{4}, \quad L = \pm \frac{1}{2}$

$2 = 2z \times \frac{1}{2} \quad z = \pm 2.$

$x^2 + y^2 + 4 = 36$

$x^2 + y^2 = 32$

$x^2 = 32 - y^2 \quad x = \sqrt{32 - y^2}$

$\sqrt{32 - y^2} = 2 \times \frac{1}{2} \cdot y$

$32 - y^2 = y^2$

$32 = 2y^2$

$y^2 = 16$

$y = \pm 4.$

$\pm 4 = 2 \times \left(\pm\frac{1}{2}\right) x$

$x = \pm 4.$

$f(\pm 4, \pm 4, 2) = 20$

$f(\pm 4, \pm 4, -2) = -20$

maximum is 20 minimum is −20

15. $f_x = y+z \quad f_y = x \quad f_z = x$

$\nabla f = \langle y+z,\ x,\ x\rangle$

$g_x = 2x \quad g_y = 2y \quad g_z = 2z$

$\nabla g = \langle 2x, 2y, 2z\rangle$

$\nabla f = L\nabla g$

$\langle y+z, x, x\rangle = L\langle 2x, 2y, 2z\rangle$

$y+z = 2Lx, \quad x = 2Ly \quad x = 2Lz$

$x^2+y^2+z^2=4$

$\cancel{x=2Ly}\ 2Ly = 2Lz$

$y = z$

$2y = 2Lx = 2L\cdot 2Ly$

$2 = 4L^2$

$\frac{2}{4} = L^2$

$L = \frac{1}{\sqrt{2}}$

$(2Ly)^2 + \left(\frac{x}{2L}\right)^2 + \left(\frac{x}{2L}\right)^2 = 4$

$\left(2L\cdot\frac{x}{2L}\right)^2 + \left(\frac{x}{2L}\right)^2 + \left(\frac{x}{2L}\right)^2 = 4$

$x^2 + \frac{x^2}{4L^2} + \frac{x^2}{4L^2} = 4$

$x^2 + \frac{x^2}{2} + \frac{x^2}{2} = 4$

$\frac{2x^2+x^2+x^2}{2} = 4$

$\frac{4x^2}{2} = 4$

$4x^2 = 8$

$x^2 = 2$

$x = \pm\sqrt{2}$

$y = \frac{x}{2L} \quad x = \sqrt{2}\ y = \sqrt{2}\cdot\frac{1}{\sqrt{2}}$

$y = 1 \quad x = -\sqrt{2} \quad y = -1$

$z = \pm 1$

$z = \frac{x}{2L} = \pm 1$

$f(\sqrt{2}, 1, 1) = \sqrt{2}\times 1 + \sqrt{2}\times 1$

$= 2\sqrt{2}$

$f(-\sqrt{2}, -1, -1) = -\sqrt{2}\times(-1) + (-\sqrt{2})\times(-1)$

$= 2\sqrt{2}$

$f(\sqrt{2}, -1, -1) = \sqrt{2}\times(-1) + (\sqrt{2})\times(-1)$

$= -\sqrt{2} - \sqrt{2}$

$= -2\sqrt{2}$

maximum is $2\sqrt{2}$

minimum is $-2\sqrt{2}$