Math 251 Shaun Goda Section 23 HW #6

January February March April May June July August September October November December
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

14.8:

5) $f(x,y) = x^2+y^2$ $g(x,y) = 2x+3y = 6$

$\nabla f(x,y) = \langle 2x, 2y \rangle$ $\nabla g(x,y) = \langle 2, 3 \rangle$

$\nabla f = \lambda \nabla g$ $2x = \lambda 2$, $2y = \lambda 3$, $2x+3y = 6$

$4xy = \lambda^2 6$ $4(\lambda)y = \lambda^2 6$ $4y = \lambda 6$ $y = \frac{3}{2}\lambda$, $x = \lambda$

$2(\lambda) + 3(\frac{3}{2}\lambda) = 6$ $2\lambda + \frac{9}{2}\lambda = 6$ $\frac{13}{2}\lambda = 6$ $\lambda = \frac{12}{13}$

$x = \frac{12}{13}$ $y = \frac{18}{13}$ $f(\frac{12}{13}, \frac{18}{13}) = (\frac{12}{13})^2 + (\frac{18}{13})^2 = \frac{36}{18}$

☆ Minimum Value: $\frac{36}{18}$

7) $f(x,y) = xy$ $g(x,y) = 4x^2+9y^2 = 32$

$\nabla f(x,y) = \langle y, x \rangle$ $\nabla g(x,y) = \langle 8x, 18y \rangle$

$y = \lambda 8x$, $x = \lambda 18y$, $4x^2+9y^2 = 32$

$xy = \lambda^2 144xy$ $x(\lambda 8x) = \lambda^2 144x(\lambda 8x)$

$1 = \lambda^2 144$ $\lambda = \sqrt{\frac{1}{144}} = \frac{1}{12}$

$y = \frac{2}{3}x$, $x = \frac{3}{2}y$

☆ min $\frac{8}{3}$ max $-\frac{8}{3}$

9) $f(x,y) = x^2+y^2$ $g(x,y) = x^4+y^4 = 1$

$\nabla f(x,y) = \langle 2x, 2y \rangle$ $\nabla g(x) = \langle 4x^3, 4y^3 \rangle$

$2x = \lambda 4x^3$, $2y = \lambda 4y^3$, $x^4+y^4 = 1$

$4xy = \lambda^2 16x^3y^3$ $1 = \lambda^2 4x^2y^2$ $\frac{1}{4} = \lambda^2 x^2 y^2$

$\frac{1}{4} = \lambda^2 x^2 (\lambda 2y^3)^2$ $\frac{1}{4} = \lambda^4$ / $1 = \lambda 2x^2$ $x = \sqrt{\frac{1}{2\lambda}}$ $y = \sqrt{\frac{1}{2\lambda}}$

$(\sqrt{\frac{1}{2\lambda}})^4 + (\sqrt{\frac{1}{2\lambda}})^4 = 1$ $\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 1$ $\frac{1}{2\lambda^2} = 1$ $\lambda = \pm\sqrt{\frac{1}{2}}$

$x = \sqrt{\frac{1}{2}} 4x^3$ $1 = \sqrt{2}x^3$ $x = 2^{5/6}/2$ $y = 2^{5/6}/2$

$f(2^{5/6}/2, 2^{5/6}/2) = 1$

$1 = -\sqrt{2}\,2x^2$ $x^2 = \sqrt{\frac{1}{-\sqrt{2}\,2}} = \frac{2^{3/4}}{2} = y$

$f(2^{3/4}/2, 2^{3/4}/2) = \sqrt{2}$

☆ minimum value: 1

☆ maximum value: $\sqrt{2}$

11) $f(x,y,z) = 3x + 2y + 4z \quad g(x,y,z) = x^2 + 2y^2 + 6z^2 = 1$

$\nabla f(x,y,z) = \langle 3, 2, 4 \rangle \nabla g(x,y,z) = \langle 2x, 4y, 12z \rangle$

$3 = 2\lambda x, \; 2 = 4\lambda y, \; 4 = 12\lambda z, \; x^2 + 2y^2 + 6z^2 = 1$

$24 = 96\lambda^3 xyz \quad 24 = 96\lambda^3 x\left(\frac{1}{2\lambda}\right)\left(\frac{1}{3\lambda}\right) \quad \frac{3}{2} = \lambda x \quad x = \frac{3}{2\lambda}$

$24 = 96\lambda^3\left(\frac{3}{2\lambda}\right)\left(\frac{1}{2\lambda}\right)z \quad z = \frac{1}{3\lambda} \quad 24 = 96\lambda^3\left(\frac{3}{2\lambda}\right)y\left(\frac{1}{3\lambda}\right) \quad y = \frac{1}{2\lambda}$

$\left(\frac{3}{2\lambda}\right)^2 + 2\left(\frac{1}{2\lambda}\right)^2 + 6\left(\frac{1}{3\lambda}\right)^2 = 1 \quad \frac{41}{12\lambda} = 1 \quad \lambda = \frac{41}{12}$

$2x\left(\frac{41}{12}\right) = 3 \quad x = \frac{18}{41}, \quad 2 = 4\left(\frac{41}{12}\right)y \quad y = \frac{1}{41}, \quad 4 = 12z\left(\frac{41}{12}\right) \quad z = \frac{4}{41}$

$3\left(\frac{18}{41}\right) + 2\left(\frac{1}{41}\right) + 4\left(\frac{4}{41}\right) = 72/41$

☆ minimum : $72/41$

13) $f(x,y,z) = xy + 2z \quad g(x,y,z) = x^2 + y^2 + z^2 = 36$

$\nabla f(x,y,z) = \langle y, x, 2 \rangle \nabla g(x,y,z) = \langle 2x, 2y, 2z \rangle$

$y = \lambda 2x, \; x = \lambda 2y, \; 2 = \lambda 2z, \; x^2 + y^2 + z^2 = 36$

$2xy = 8\lambda^3 xyz \quad 1 = 4z\lambda^3 \quad z = \frac{1}{4\lambda^3}$

$2 = \lambda 2\frac{1}{4\lambda^3} \quad 1 = \lambda^{-2} \quad \lambda = \pm 1 \quad z = \pm 2$

$y = \lambda 2(\lambda 2y) \quad 1 = \lambda^2 2 \quad y = \pm 4 \quad x, 4$

$f(4,4,2) = 20 \quad f(4,4,-2) = 12$

☆ Maximum : 20

☆ Minimum : 12

15) $f(x,y,z) = xy + xz \quad g(x,y,z) = x^2 + y^2 + z^2 = 4$

$\nabla f(x,y,z) = \langle y + z, x, x \rangle \nabla g(x,y,z) = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g \quad y + z = \lambda 2x, \; x = \lambda 2y, \; x = \lambda 2z$

$x^2(y+z) = \lambda^3 8xyz$

$x^2(\lambda 2x) = \lambda^3 8xyz \quad x^2 = \lambda^2 4yz \quad x^2 = x\lambda 2z$

$x^2(y + \lambda 2x) = \lambda^2 4yz \quad x^3 = \lambda 2yz,$

I don't know how to solve this problem.
What do I do when I got $x = x$ or $y = y$?