HW due 10/18/20

14.8 5, 7, 9, 11, 13, 15

5. $f(x,y) = x^2+y^2 \quad 2x+3y=6$

$\nabla f = \langle 2x, 2y\rangle \quad \nabla g = \langle 2, 3\rangle$

$\langle 2x, 2y\rangle = \lambda\langle 2, 3\rangle$

$2x = 2\lambda \quad 2y = 3\lambda$

$x = \lambda \quad \lambda = \frac{2y}{3} = x$

$2\left(\frac{2y}{3}\right) + 3y = 6$

$\frac{4y}{3} + \frac{9y}{3} = 6 \quad \frac{13y}{3} = 6$

$13y = 18 \quad y = \frac{18}{13} \quad x = \frac{12}{13}$

$f\left(\frac{12}{13}, \frac{18}{13}\right) = \left(\frac{12}{13}\right)^2 + \left(\frac{18}{13}\right)^2 = \boxed{\frac{468}{169}}$ ← min

7. $f(x,y) = xy \quad 4x^2+9y^2 = 32$

$\nabla f = \langle y, x\rangle \quad \nabla g = \langle 8x, 18y\rangle$

$\langle y, x\rangle = \lambda\langle 8x, 18y\rangle$

$y = 8\lambda x \quad x = 18\lambda y$

$x = \frac{y}{8\lambda} \quad \frac{y}{8\lambda} = 18\lambda y \quad \lambda^2 = \frac{1}{(8)(18)}$

$\lambda = \frac{1}{12} \quad x = \frac{3}{2}y$

$4\left(\frac{3}{2}y\right)^2 + 9y^2 = 32$

$9y^2 + 9y^2 = 32 \quad 18y^2 = 32$

$y^2 = \frac{32}{18} \quad y = \frac{4\sqrt{2}}{3\sqrt{2}} = \frac{4}{3} \quad x = 2, -2$

$(-2)\left(\frac{4}{3}\right) = \boxed{-\frac{8}{3}}$ ← min $\quad (2)\left(\frac{4}{3}\right) = \boxed{\frac{8}{3}}$ ← max

9. $f(x,y) = x^2+y^2 \quad x^4+y^4 = 1$

$\nabla f = \langle 2x, 2y\rangle \quad \nabla g = \langle 4x^3, 4y^3\rangle$

$\langle 2x, 2y\rangle = \lambda\langle 4x^3, 4y^3\rangle$

$2x = 4\lambda x^3 \quad 2y = 4\lambda y^3$

$\frac{1}{2x^2} = \lambda \quad \frac{1}{2y^2} = \lambda \quad x = y$

$x^4 + x^4 = 1 \quad 2x^4 = 1 \quad x^4 = \frac{1}{2}$

$x = \pm\frac{1}{2^{\frac{1}{4}}} \quad y = \pm\frac{1}{2^{\frac{1}{4}}}$

critical points: $(0, \pm 1)(\pm 1, 0)\left(\pm\frac{1}{2^{\frac{1}{4}}}, \pm\frac{1}{2^{\frac{1}{4}}}\right)$

$1^2 + 0^2 = \boxed{1}$ ← max $\quad \left(\frac{1}{2^{\frac{1}{4}}}\right)^2 + \left(\frac{1}{2^{\frac{1}{4}}}\right)^2 = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \boxed{\sqrt{2}}$ ↑ min

11. $f(x,y,z) = 3x+2y+4z \quad x^2+2y^2+6z^2 = 1$

$\nabla f = \langle 3, 2, 4\rangle \quad \nabla g = \langle 2x, 4y, 12z\rangle$

$\langle 3, 2, 4\rangle = \lambda\langle 2x, 4y, 12z\rangle$

$3 = 2\lambda x \quad 2 = 4\lambda y \quad 4 = 12\lambda z$

$\lambda = \frac{3}{2x} \quad \lambda = \frac{1}{2y} \quad \lambda = \frac{1}{3z}$

$2x = 6y \quad x = 3y \quad 3z = 2y \quad z = \frac{2}{3}y$

$(3y)^2 + 2y^2 + 6\left(\frac{2}{3}y\right)^2 = 1$

$9y^2 + 2y^2 + \frac{8}{3}y^2 = 1$

$\frac{41}{3}y^2 = 1 \quad y^2 = \frac{3}{41} \quad y = \pm\sqrt{\frac{3}{41}}$

$x = \pm\frac{3\sqrt{3}}{\sqrt{41}} \quad z = \pm\frac{2\sqrt{3}}{3\sqrt{41}}$

max: $\frac{9\sqrt{3}}{\sqrt{41}} + \frac{2\sqrt{3}}{\sqrt{41}} + \frac{8\sqrt{3}}{3\sqrt{41}} = \frac{41\sqrt{3}}{3\sqrt{41}}$

min: $-\frac{41\sqrt{3}}{3\sqrt{41}}$

13. $f(x,y,z) = xy+2z \quad x^2+y^2+z^2 = 36$

$\nabla f = \langle y, x, 2\rangle \quad \nabla g = \langle 2x, 2y, 2z\rangle$

$\langle y, x, 2\rangle = \lambda\langle 2x, 2y, 2z\rangle$

$y = 2\lambda x \quad x = 2\lambda y \quad 2 = 2\lambda z$

$y = 4\lambda^2 y \quad 1 = 4\lambda^2 \quad \lambda = \pm\frac{1}{2} \quad z = \pm 2 \quad x = y$

$2y^2 + 4 = 36 \quad y^2 = 16 \quad y = \pm 4 \quad x = \pm 4$

max: $(4)(4) + 2(2) = \boxed{20}$

min: $(-4)(4) + 2(-2) = \boxed{-20}$

15. $f(x,y,z) = xy+xz \quad x^2+y^2+z^2 = 4$

$\nabla f = \langle y+z, x, x\rangle \quad \nabla g = \langle 2x, 2y, 2z\rangle$

$\langle y+z, x, x\rangle = \lambda\langle 2x, 2y, 2z\rangle$

$y+z = 2\lambda x \quad x = 2\lambda y \quad x = 2\lambda z \quad y = z$

$\frac{y+z}{2x} = \frac{x}{2y} \quad x^2 = y(y+z) \quad x^2 = 2y^2$

$y = \pm 1 \quad z = \pm 1 \quad x = \pm\sqrt{2}$

max $(\sqrt{2})(1) + (\sqrt{2})(1) = \boxed{2\sqrt{2}}$

min: $(\sqrt{2})(-1) + (\sqrt{2})(-1) = \boxed{-2\sqrt{2}}$