14.8 (Oct. 18th)

14.8: #5, 7, 9, 11, 13, 15

5) $f(x,y) = x^2+y^2$

$g(x,y) = 2x+3y-6$

$\nabla f = \langle 2x, 2y \rangle$ , $\nabla g = \langle 2, 3 \rangle$

Lagrange Equations $\nabla f_p = \lambda \nabla g_p$

$\langle 2x, 2y \rangle = \lambda \langle 2, 3 \rangle \rightarrow 2x = 2\lambda \quad 2y = 3\lambda$

$\lambda = x$ and $3\lambda = 2y$

$\lambda = x = \frac{2y}{3}$

$2y = 3x$ , $y = \frac{3}{2}x$

$2x+3y-6=0 \rightarrow 2x+3\left(\frac{3}{2}x\right)-6=0$

$\frac{13}{2}x = 6$ , $x = \frac{12}{13}$

$P = \left(\frac{12}{13}, \frac{18}{13}\right)$

$f(P) = f\left(\frac{12}{13}, \frac{18}{13}\right) = \frac{36}{13}$

No max, min is $\frac{36}{13}$

7) $f(x,y) = xy$

$g(x,y) = 4x^2+9y^2-32$

$\nabla g = \langle 8x, 18y \rangle$ , $\nabla f = \langle y, x \rangle$

$\langle y, x \rangle = \lambda \langle 8x, 18y \rangle \rightarrow y = \lambda(8x)$ , $x = \lambda(18y)$

$y = 8\lambda x$ , $x = 18\lambda y$

$\lambda = \frac{y}{8x} = \frac{x}{18y}$

$2x = \pm 3y$

$4x^2 + 4x^2 = 32$

$x = \pm 2$ , since $y = \frac{2}{3}x$ , cp are $\left(\pm 2, \pm \frac{4}{9}\right)$

$f(2, \frac{4}{3}) = f(-2, -\frac{4}{3}) = \frac{8}{3}$

$f(-2, \frac{4}{3}) = f(2, -\frac{4}{3}) = -8/3$

max is $\frac{8}{3}$, min is $-\frac{8}{3}$

9) $f(x,y) = x^2+y^2$

$g(x,y) = x^4+y^4-1$

$\nabla f = \langle 2x, 2y \rangle$, $\nabla g = \langle 4x^3, 4y^3 \rangle$

$\langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle \rightarrow 2x = 4x^3\lambda$, $2y = 4y^3\lambda$

$2y = 4y^3\lambda \rightarrow y(1-2y^2\lambda) = 0$

$\lambda = \frac{1}{2x^2} = \frac{1}{2y^2}$

$x^4+y^4-1=0 \rightarrow x^4+x^4-1=0$

$2x^4 = 1$

$x = \pm \frac{1}{(2)^{1/4}}$

8 cp: $(\pm \frac{1}{2^{1/4}}, \pm \frac{1}{2^{1/4}})$, $(\pm 1, 0)$ and $(0, \pm 1)$

$f(\pm \frac{1}{2^{1/4}}, \pm \frac{1}{2^{1/4}}) = \sqrt{2}$

$f(\pm 1, 0) = f(0, \pm 1) = 1$

max is $\sqrt{2}$, min is 1

11) $f(x,y,z) = 3x+2y+4z$

$g(x,y,z) = x^2+2y^2+6z^2-1$

$\nabla f = \langle 3, 2, 4 \rangle$, $\nabla g = \langle 2x, 4y, 12z \rangle$

$\langle 3, 2, 4 \rangle = \lambda \langle 2x, 4y, 12z \rangle \rightarrow 3 = \lambda(2x)$, $2 = \lambda(4y)$, $4 = \lambda(12z)$

$2\lambda x = 3$, $\lambda(2y) = 1$, and $\lambda(3z) = 1$

$\lambda = \frac{3}{2x} = \frac{1}{2y} = \frac{1}{3z}$

$y = x/3$, $z = 2x/9$

$x^2 + \left(\frac{x}{3}\right)^2 + \left(\frac{2x}{9}\right)^2 - 1 = 0$

$x = \pm \frac{9}{\sqrt{123}}$

$P = \left(\frac{9}{\sqrt{123}}, \frac{3}{\sqrt{123}}, \frac{2}{\sqrt{123}}\right)$, $Q = \left(-\frac{9}{\sqrt{123}}, \frac{-3}{\sqrt{123}}, \frac{-2}{\sqrt{123}}\right)$

$f(P) = \frac{41}{\sqrt{123}} = 3.7$

$f(Q) = \frac{-41}{\sqrt{123}} = -3.7$

**max is 3.7, min is -3.7**

13) $f(x,y,z) = xy + z^2$

$g(x,y,z) = x^2 + y^2 + z^2 = 36$

$\nabla f = \langle y, x, 2z \rangle$, $\nabla g = \langle 2x, 2y, 2z \rangle$

$\langle y, x, 2z \rangle = \lambda \langle 2x, 2y, 2z \rangle \rightarrow y = \lambda(2x),\ x = \lambda(2y),\ 2z = \lambda(2z)$

$2\lambda x = y$, $2\lambda(y) = x$, $\lambda z = 1$

$2\lambda x = y$, $2\lambda(y) = x \rightarrow y = 2\lambda(2\lambda y)$

$y(1 - 4\lambda^2) = 0$

$y = 0 \rightarrow x = 0$, $z = \pm 6$

$2\lambda = 1$, so $z = \pm 2$ and $x = \pm y$

$x^2 + (\pm x)^2 + (\pm 2)^2 - 36 = 0$

Answer

$x = \pm 4$

CP: $(\pm 4, \pm 4, 2)$ and $(0, 0, \pm 6)$

$f(4,4,2) = f(-4,-4,2) = 20$

$f(4,-4,-2) = f(-4,4,-2) = -20$

$f(4,4,-2) = f(-4,-4,-2) = 12$

$f(4,-4,2) = f(-4,4,2) = 12$

$f(0,0,\pm 6) = \pm 12$

**Maxima at $f(\pm 4, \pm 4, 2) = 20$**

**minima at $f(\pm 4, \pm 4, -2) = -20$**

15) $f(x,y,z) = xy + xz$

$g(x,y,z) = x^2 + y^2 + z^2 - 4$

$\nabla f = \langle y+z, x, x \rangle$, $\nabla g = \langle 2x, 2y, 2z \rangle$

$\langle y+z, x, x \rangle = \lambda \langle 2x, 2y, 2z \rangle \rightarrow y+z = \lambda(2x)$, $x = \lambda(2y)$, $x = \lambda(2z)$

$x = 2\lambda y = 2\lambda z$ and $2\lambda x = y+z$

$x^2 + y^2 + z^2 - 4 = 0 \rightarrow 2y^2 = 4$, $y = \pm\sqrt{2}$

$2\lambda = \frac{y+z}{x}$, $\frac{2\lambda}{x} = y = z \rightarrow 2\lambda = \frac{y+z}{x} = \frac{x}{y}$, $y = z$

$\frac{2y}{x} = \frac{x}{y}$

$x^2 = 2y^2$

$2y^2 + y^2 + y^2 - 4 = 0$

$y = \pm 1$

CP = $(\pm\sqrt{2}, 1, 1)$ and $(\pm\sqrt{2}, -1, -1)$ and $(0, \pm\sqrt{2}, \pm\sqrt{2})$

$f(\pm\sqrt{2}, 1, 1) = 2\sqrt{2}$

$f(\pm\sqrt{2}, -1, -1) = -2\sqrt{2}$

$f(0, 0, \pm\sqrt{2}) = 0$

max is $2\sqrt{2}$ at $(\pm\sqrt{2}, 1, 1)$

min is $-2\sqrt{2}$ at $(\pm\sqrt{2}, -1, -1)$