HW → Section 14.8
Due 10/18

Rahul Paleja

14.8 → #5, 7, 9, 11, 13, 15:

(5) $f(x,y) = x^2+y^2$, $2x+3y=6$

$Df = \langle 2x, 2y \rangle$ $Dg = \langle 2, 3 \rangle$

$Df = \lambda Dg$

$\frac{2x}{2} = \frac{\lambda 2}{2}$ $\frac{2y}{3y} = \frac{\lambda 3}{3}$

$x = \lambda$ $\frac{2}{3} = \lambda$ $\frac{2y}{3} = x$ $y = \frac{3x}{2}$

$2x + 3\left(\frac{3x}{2}\right) = 6$

$\frac{2x}{1} + \frac{9x}{2} = 6 \rightarrow \frac{13x}{2} = 6$ $13x = 12$ $x = \frac{12}{13}$

$y = \frac{3}{2}\left(\frac{12}{13}\right) = \frac{36}{26} = \frac{18}{13}$

Point $\left(\frac{12}{13}, \frac{18}{13}\right)$

$f\left(\frac{12}{13}, \frac{18}{13}\right) = \left(\frac{12}{13}\right)^2 + \left(\frac{18}{13}\right)^2 = \frac{468}{169} = \frac{36}{13}$

Min value → $\frac{36}{13}$, No max

(7) $f(x,y) = xy$ $4x^2+9y^2=32$

$Df = \langle y, x \rangle$ $Dg = \langle 8x, 18y \rangle$

$Df = \lambda Dg$

$y = \lambda 8x$ $x = \lambda 18y$

$\lambda = \frac{y}{8x} = \frac{x}{18y}$ $18y^2 = 8x^2$ $9y^2 = 4x^2$

$\pm 3y = \pm 2x$

$y = \pm \frac{2}{3}x$

$4x^2 + 9\left(\frac{2}{3}x\right)^2 = 32$

$4x^2 + 4x^2 = 32$ $8x^2 = 32$ $x^2 = 4$ $x = \pm 2$

Critical Points: $(2, 4/3)$, $(-2, -4/3)$ $(2, -4/3)$ $(-2, 4/3)$

$f(2, 4/3) = 8/3$
$f(-2, -4/3) = 8/3$
$f(2, -4/3) = -8/3$
$f(-2, 4/3) = -8/3$

Max Value $= 8/3$
Min Value $= -8/3$

(9) $f(x,y) = x^2 + y^2 \qquad x^4 + y^4 = 1$

$\nabla f = \langle 2x, 2y \rangle \qquad \nabla g = \langle 4x^3, 4y^3 \rangle$

$x = 0 \qquad 2x = \lambda 4x^3 \qquad 2y = \lambda 4y^3 \qquad y = 0 \qquad 0 = \lambda 4y^3 - 2y = \lambda(4y^2 - 2)$

$0 = x(4\lambda x^3 - 2x) \quad \frac{1}{2x^2} = \lambda \qquad \frac{1}{2y^2} = \lambda \qquad \frac{1}{2x^2} = \frac{1}{2y^2} \qquad x^2 = y^2$

$0 = \lambda(4x^2 - 2) \qquad\qquad\qquad\qquad\qquad\qquad y = \pm x$

$x^4 + x^4 = 1 \qquad 2x^4 = 1 \qquad x^4 = \frac{1}{2} \qquad x = \pm \frac{1}{(2)^{1/4}}$

Critical Point:

$\left(\frac{1}{2^{1/4}}, \frac{1}{2^{1/4}}\right), \left(\frac{-1}{2^{1/4}}, \frac{-1}{2^{1/4}}\right), \left(\frac{1}{2^{1/4}}, \frac{-1}{2^{1/4}}\right), \left(\frac{-1}{2^{1/4}}, \frac{1}{2^{1/4}}\right)$

$(\pm 1, 0) \qquad (0, \pm 1)$

$f\left(\pm \frac{1}{2^{1/4}}, \pm \frac{1}{2^{1/4}}\right) = \sqrt{2}$

$f(\pm 1, 0) = 1$

$f(0, \pm 1) = 1$

Max Value $= \sqrt{2}$
Min value $= 1$

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14.8 → #11, 13, 15:

(11) $f(x,y,z) = 3x+2y+4z$ $\quad x^2+2y^2+6z^2=1$

$Df = \langle 3,2,4\rangle$ $\quad Dg = \langle 2x, 4y, 12z\rangle$

$Df = \lambda Dg$

$3 = \lambda 2x$ $\quad 2 = \lambda 4y$ $\quad 4 = \lambda 12z$

$\frac{3}{2x} = \lambda$ $\quad \frac{1}{2y} = \lambda$ $\quad \frac{1}{3z} = \lambda$

$\frac{3}{2x} = \frac{1}{2y}$ $\quad 6y = 2x$ $\quad \frac{3}{2x} = \frac{1}{3z}$

$y = \frac{x}{3}$ $\quad 9z = 2x$ $\quad z = \frac{2}{9}x$

$x^2 + 2\left(\frac{x}{3}\right)^2 + 6\left(\frac{2}{9}x\right)^2 = 1$

$x^2 + 2\left(\frac{x^2}{9}\right) + 6\left(\frac{4x^2}{81}\right) = 1$

$x^2 + \frac{2x^2}{9} + \frac{24x^2}{81} = 1$ $\quad \frac{81x^2}{81} + \frac{42x^2}{81} = 1$

$\frac{123x^2}{81} = 1$ $\quad 123x^2 = 81$ $\quad x^2 = \frac{81}{123}$ $\quad x = \pm\frac{9}{\sqrt{123}}$

Critical Points:

$f\left(\frac{9}{\sqrt{123}}, \frac{3}{\sqrt{123}}, \frac{2}{\sqrt{123}}\right) = \frac{41}{\sqrt{123}}$

$f\left(\frac{-9}{\sqrt{123}}, \frac{-3}{\sqrt{123}}, \frac{-2}{\sqrt{123}}\right) = \frac{-41}{\sqrt{123}}$

Max value $= \frac{41}{\sqrt{123}}$

Min value $= \frac{-41}{\sqrt{123}}$

(13) $f(x,y,z) = xy + 2z$ $\quad x^2+y^2+z^2 = 36$

$Df = \langle y, x, 2 \rangle$ $\quad Dg = \langle 2x, 2y, 2z \rangle$

$y = \lambda 2x$ $\quad x = \lambda 2y$ $\quad 2 = \lambda 2z$

$\frac{y}{2x} = \lambda$ $\quad \frac{x}{2y} = \lambda$ $\quad \frac{1}{2} = \lambda$ $\quad \frac{1}{2} = \frac{y}{2x}$ $\pm x$

$\frac{y}{2x} = \frac{x}{2y}$ $\quad 2y^2 = 2x^2$ $\quad 2x = 2y$

$y^2 = x^2$ $\quad y = \pm x$ $\quad \frac{2x}{x} = \frac{xz}{x}$ $\quad z = \pm 2$

$x^2 + (\pm x)^2 + (\pm 2)^2 = 36$

$2x^2 + 4 = 36$ $\quad 2x^2 = 32$ $\quad x^2 = 16$ $\quad x = \pm 4$

Critical Pts:

$f(4,4,2) = 20$

$f(-4,4,2) = -20$

$f(-4,-4,2) = 20$

$f(4,-4,2) = -12$

$f(4,4,-2) = 12$

$f(-4,-4,-2) = 12$

$f(4,-4,-2) = -20$

$f(-4,4,-2) = -20$

Min value $= -20$
Max value $= 20$

(15) $F(x,y,z) = xy + xz$, $x^2+y^2+z^2 = 4$

$Df = \langle y+z, x, x \rangle$ $\quad Dg = \langle 2x, 2y, 2z \rangle$

$y+z = \lambda 2x$ $\quad x = \lambda 2y$ $\quad x = \lambda 2z$

$\frac{y+z}{2x} = \lambda$ $\quad \frac{x}{2y} = \lambda$ $\quad \frac{x}{2z} = \lambda$ $\quad \frac{x}{2y} = \frac{x}{2z}$ $\quad 2yx = 2zx$

$y = z$

$\frac{y}{x} = \frac{x}{2y}$ $\quad 2y^2 = x^2$

$2y^2 + y^2 + y^2 = 4$ $\quad 4y^2 = 4$ $\quad y^2 = 1$ $\quad y = \pm 1$

Critical Points: $f(\pm\sqrt{2}, 1, 1) = 2\sqrt{2}$

$f(\pm\sqrt{2}, -1, -1) = -2\sqrt{2}$

Max value $2\sqrt{2}$
Min value $-2\sqrt{2}$