Rachel Balji
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Midcom

WÜRTH ELEKTRONIK

14.8 Homework: 5, 7, 9, 11, 13, 15

(5) $f(x,y) = x^2 + y^2$ $\quad 2x+3y=6$

$f_x = 2x$, $f_y = 2y$, $f_z = 0$ $\Rightarrow \nabla f = \langle 2x, 2y, 0\rangle$

$g_x = 2$, $g_y = 3$, $g_z = 0$ $\Rightarrow \nabla g = \langle 2, 3, 0\rangle$

$\nabla f = \lambda \nabla g$

$\langle 2x, 2y, 0\rangle = \lambda \langle 2, 3, 0\rangle$

$2x = 2\lambda \longrightarrow 2x = 2\lambda$

$2y = 3\lambda$

$0 = 0$

$x = \lambda$

$2y = 3\lambda$

$y = 3/2\,\lambda$

$2(\lambda) + 3(3/2)\lambda = 6$

$\frac{13}{2}\lambda = 6 \Rightarrow \lambda = \frac{12}{13}$

$x = \frac{12}{13}$ $\quad y = \frac{36}{26}$

$f(x,y) = \frac{(12)^2}{13^2} + \left(\frac{36}{26}\right)^2$

$f\left(\frac{12}{13}, \frac{36}{26}\right) = 2.769 \approx 2.77$ * minimum (no max value)

(11) $f(x,y,z) = 3x + 2y + 4z$ $\quad x^2 + 2y^2 + 6z^2 = 1$

$f_x = 3$, $f_y = 2$, $f_z = 4$

$g_x = 2x$, $g_y = 4y$, $g_z = 12z$

$\langle 3, 2, 4\rangle = \lambda \langle 2x, 4y, 12z\rangle$

$3 = \lambda 2x \longrightarrow x = \frac{3}{2\lambda}$

$2 = \lambda 4y \quad y = \frac{1}{2\lambda}$

$4 = 12z\lambda \quad z = \frac{1}{3\lambda}$

$\left(\frac{9}{4\lambda^2}\right) + 2\left(\frac{1}{4\lambda^2}\right) + \left(\frac{6}{9\lambda^2}\right) = 1$

$\frac{11}{4\lambda^2} + \frac{6}{9\lambda^2} = 1$

$\frac{99}{4\lambda^2} + \frac{24}{9\lambda^2} = 1 \Rightarrow \frac{123}{36\lambda^2} = 1$

$\lambda = \pm\frac{\sqrt{123}}{6}$

$x = 0.812$

$f(x,y,z) = y = 0.271$

$z = 0.18$

$f(x,y,z) = 3(0.812) + 2(0.271) + 4(0.18)$

$= 3.7 = \max$

$(-3.7 = \min)$

(7) $f(x,y) = xy$ $\quad 4x^2 + 9y^2 = 32$

$f_x = y$, $f_y = x$ $\quad \nabla f = \lambda \nabla g$ $\quad \langle y, x\rangle = \lambda \langle 8x, 18y\rangle$

$g_x = 8x$, $g_y = 18y$

$y = \lambda 8x \longrightarrow y = \frac{2}{3}x$

$x = \lambda 18y \longrightarrow x = \frac{3}{2}y$

$x = \lambda^2 144x$

$1 = \lambda^2 144$

$\frac{1}{144} = \lambda^2 \Rightarrow \lambda = \frac{1}{12}$

$y = \frac{2}{3}\cdot\frac{3}{2}y$

$y = y$

(9) $f(x,y) = x^2 + y^2$ $\quad x^4 + y^4 = 1$ $\quad x = 1$, $y = 1$ minimum at 1

$f_x = 2x$, $f_y = 2y$ $\quad g_x = 4x^3$, $g_y = 4y^3$ $\quad \max = \sqrt{2}$

$\langle 2x, 2y\rangle = \lambda \langle 4x^3, 4y^3\rangle$

$2x = 4x^3\lambda$

$2y = 4y^3\lambda$

$x = 2x^3\lambda$

$\frac{1}{2\lambda} = x^2$

$y = \pm\frac{1}{\sqrt{2\lambda}}$

$\pm\frac{1}{\sqrt{2\lambda}} = x$

$\frac{1}{2(2\lambda)} + \frac{1}{2(2\lambda)} = 1$

$\frac{2}{4\lambda} = 1 \quad \lambda = \frac{1}{2}$

(13) ?

$f(x,y,z) = xy + 2z \qquad x^2+y^2+z^2 = 36$

$f_x = y$
$f_y = x$
$f_z = 2$

$g_x = 2x$
$g_y = 2y$
$g_z = 2z$

$y = \lambda 2x \Rightarrow x = 2\lambda y$
$2 = 2z\lambda$
$z = 1\lambda \Rightarrow z = \lambda$

$xyz = \lambda^3 4x\lambda$
$yz = \lambda^3$
$y = \lambda^3 z \Rightarrow y = \lambda^4$

$x = 2\lambda y$
$x = 2\lambda\lambda 2x$
$1 = \lambda^2$
$\lambda = \pm 1$

$y = 2x \qquad y = 4y$
$x = 2y$
$z = 1$

$= 2y(2x) + 2$

$x^2 + 4x^2 + 1 = 36$
$5x^2 = 35$
$x^2 = \pm 7$
$x = \pm\sqrt{7}$
$(\sqrt{7}, 16)$

$y = 16$

$x^2+y^2+z^2 = 36$
$4\lambda^2 + \lambda^8 + \lambda^2 = 36$
$5\lambda^2 + \lambda^8 = 36$
$\lambda^2(5+\lambda^6) = 36$
$\lambda = -1.4941$ and 1.4941

$x = 2(1.4941) = 2.9828$
$y = (1.4941)^4 = 4.94739$
$z = 1.4914$

(15)

$f(x,y,z) = xy + xz \qquad x^2+y^2+z^2 = 4$

$f_x = y + 2x$
$f_y = x$
$f_z = x$

$g_x = 2x$
$g_y = 2y$
$g_z = 2z$

$y + 2x = 2x\lambda$
$x = 2y\lambda$
$x = 2z\lambda$

$y + 2(2y\lambda) = 2(2y\lambda)\lambda$
$y + 4y\lambda = 4\lambda^2 y$
$y(1 + 4\lambda) = 4\lambda^2 y$
$1 + 4\lambda = 4\lambda^2$
$1 = 4\lambda^2 - 4\lambda$
$1 = 4\lambda(\lambda - 1)$
$\lambda = 1.207$ and -0.207

$y + 2x = 2.4x$
$y = 0.4x$

$x = 0.8x(1.2)$
$1x = 0.96x$