14.8 Homework

5) $f(x,y) = x^2 + y^2$, $g(x,y) = 2x + 3y = 6$

$f_x = 2x$, $f_y = 2y$, $g_x = 2$, $g_y = 3$

$\nabla f = \langle 2x, 2y \rangle$, $\nabla g = \langle 2, 3 \rangle$

$\nabla f = \lambda \nabla g$

$\langle 2x, 2y \rangle = \lambda \langle 2, 3 \rangle$

$2x = 2\lambda$, $2y = 3\lambda$

$x = \lambda$, $y = \frac{3}{2}\lambda$, $2x + 3y = 6$

$2\lambda + \frac{9}{2}\lambda = 6$

$4\lambda + 9\lambda = 12$

$13\lambda = 12$

$\lambda = \frac{12}{13}$

Critical Point @ $(\frac{12}{13}, \frac{18}{13})$

$f(x,y) = (\frac{12}{13})^2 + (\frac{18}{13})^2$

$f(x,y) = \frac{36}{13}$

No max, $\frac{36}{13}$ is the min

7) $f(x,y) = xy$, $4x^2 + 9y^2 = 32$

$f_x = y$, $f_y = x$, $g_x = 8x$, $g_y = 18y$

$\nabla f = \langle y, x \rangle$, $\nabla g = \langle 8x, 18y \rangle$

$\nabla f = \lambda \nabla g$

$\langle y, x \rangle = \lambda \langle 8x, 18y \rangle$

$y = 8\lambda x$, $x = 18\lambda y$

$\lambda = \frac{y}{8x}$, $\lambda = \frac{x}{18y}$

$\frac{y}{8x} = \frac{x}{18y^2}$

$8x^2 = 18y^2$

$4x^2 = 9y^2$

$4x^2 + 4x^2 = 32$

$8x^2 = 32$

$x = \pm 2$

$4x^2 = 9y^2$

$16 = 9y^2$

$y = \pm \frac{4}{3}$

Critical Points $(\pm 2, \pm \frac{4}{3})$

$f(x,y) = xy$

$f(-2, -\frac{4}{3}) = \frac{8}{3}$

$f(2, \frac{4}{3}) = \frac{8}{3}$

$f(-2, \frac{4}{3}) = -\frac{8}{3}$

$f(2, -\frac{4}{3}) = -\frac{8}{3}$

Max is $\frac{8}{3}$ and Min is $-\frac{8}{3}$

9) $f(x,y) = x^2 + y^2$, $x^4 + y^4 = 1$

$f_x = 2x$, $f_y = 2y$, $g_x = 4x^3$, $g_y = 4y^3$

$\nabla f_x = \langle 2x, 2y \rangle$, $\nabla g = \langle 4x^3, 4y^3 \rangle$

$\nabla f = \lambda \nabla g$

$\langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle$

$2x = 4\lambda x^3$, $2y = 4\lambda y^3$

$1 = 2\lambda x^2$, $1 = 2\lambda y^2$

$\lambda = \frac{1}{2x^2}$, $\lambda = \frac{1}{2y^2}$

$2x^2 = 2y^2$

$x^2 = y^2$

$(x^2)^2 + (x^2)^2 = 1$

Critical Points @ $(\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}})$, $(0, \pm 1)$, $(\pm 1, 0)$

$f(x,y) = x^2 + y^2$

$f(\pm \frac{1}{\sqrt{2}}, \pm \frac{1}{\sqrt{2}}) = \frac{1}{2} + \frac{1}{2} = \frac{2}{\sqrt{2}}$

$f(0, -1) = 1$

$f(0, 1) = 1$

$f(-1, 0) = 1$

$f(1, 0) = 1$

Min is 1, Max is $\frac{2}{\sqrt{2}}$

11) $f(x,y,z) = 3x + 2y + 4z$, $x^2 + 2y^2 + 6z^2 = 1$

$f_x = 3$, $f_y = 2$, $f = 4$, $g_x = 2x$, $g_y = 4y$, $g_z = 12z$

$\nabla f = \langle 3, 2, 4 \rangle$, $\nabla g = \langle 2x, 4y, 12z \rangle$

$\nabla f = \lambda \nabla g$

$\langle 3, 2, 4 \rangle = \lambda \langle 2x, 4y, 12z \rangle$

$3 = 2\lambda x$, $1 = 2\lambda y$, $1 = 3\lambda z$

$\lambda = \frac{3}{2x}$, $\lambda = \frac{1}{2y}$, $\lambda = \frac{1}{3z}$

Critical Points: $(\pm 3\sqrt{\frac{3}{41}}, \pm \sqrt{\frac{3}{41}}, \pm \frac{2}{\sqrt{123}})$

Max is $\sqrt{\frac{41}{3}}$ and Min is $-\sqrt{\frac{41}{3}}$

13) $f(x,y) = xy + 2z$, $x^2 + y^2 + z^2 = 36$

$f_x = y$, $f_y = x$, $f_z = 2$, $g_x = 2x$, $g_y = 2y$, $g_z = 2z$

$\nabla f = \langle y, x, 2 \rangle$, $\nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g$

$\langle y, x, z \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$y = 2\lambda x$, $x = 2\lambda y$, $z = 2\lambda z$

$\lambda = \frac{y}{2x} = \frac{x}{2y} = \frac{1}{2}$

Critical Points @ $(\pm 4, \pm 4, 2)$

Max is 20, Min is -20

15) $f(x,y,z) = xy + xz$, $x^2 + y^2 + z^2 = 4$

$f_x = y + z$, $f_y = x$, $f_z = x$, $g_x = 2x$, $g_y = 2y$, $g_z = 2z$

$\nabla f = \langle y+z, x, x \rangle$, $\nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g$

$\langle y+z, x, x \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$y + z = 2\lambda x$, $1 = 2\lambda$, $x = 2z$

Critical Points @ $(\pm \sqrt{2}, \pm 1, \pm 1)$

Max is $2\sqrt{2}$ and Min is $-2\sqrt{2}$