14.8: 5 7 9 11 13 15

Chapter 14 HW

14.8

5) $f(x,y) = x^2 + y^2$, $g(x) = 2x + 3y = 6$

$\nabla f = \langle 2x, 2y \rangle$ $\nabla g = \langle 2, 3 \rangle$ $\nabla f = \lambda \nabla g$

$2x = 2\lambda$ $2y = 3\lambda$

$x = \lambda$ $y = \frac{3}{2}\lambda$

$2(\lambda) + 3\left(\frac{3\lambda}{2}\right) = 6$

$\lambda = \frac{12}{13}$

$x = \frac{12}{13}$ $y = \frac{18}{13}$

$f_{max} = \left(\frac{12}{13}\right)^2 + \left(\frac{18}{13}\right)^2$

7) $f(x,y) = xy$, $g(x,y) = 4x^2 + 9y^2 = 32$

$\nabla f = \langle y, x \rangle$ $\nabla g = \langle 8x, 18y \rangle$

$8x = \lambda y$

$18y = \lambda x$ $\Rightarrow$ $144xy = \lambda^2 xy$ $\lambda = \pm 12$

$4x^2 + 9y^2 = 32$

$9y^2 + 9y^2 = 32$

$18y^2 = 32$ $y = \pm\sqrt{\frac{16}{9}} = \pm\frac{4}{3}$

$4x^2 + 9\left(\frac{16}{9}\right) = 32$

$4x^2 = 16$ $x = \pm 2$

$8x = 12y$ $18y = 12x$ $x = \frac{3y}{2}$

$8x = -12y$ $18y = -12x$ $x = -\frac{3y}{2}$

14.8 Cont

7) $y = \pm\frac{4}{3} \quad x = \pm 2$

Max	Min
$f(2, \frac{4}{3}) = \frac{8}{3}$	$f(-2, \frac{4}{3}) = -\frac{8}{3}$
$f(2, -\frac{4}{3}) = \frac{8}{3}$	$f(2, -\frac{4}{3}) = -\frac{8}{3}$

$f(2, \frac{4}{3}) = \frac{8}{3}$

$f(2, -\frac{4}{3}) = -\frac{8}{3}$

$f(-2, +\frac{4}{3}) = -\frac{8}{3}$

$f(-2, -\frac{4}{3}) = \frac{8}{3}$

9) $f(x, y) = x^2 + y^2 \quad g(x, y) = x^4 + y^4 = 1$

$\nabla f = \langle 2x, 2y \rangle \quad \nabla g = \langle 4x^3, 4y^3 \rangle \quad \nabla g = \lambda \nabla f$

$4x^3 = 2x\lambda$

$4y^3 = 2y\lambda$

$x^4 + y^4 = 1 \Rightarrow \frac{\lambda^2}{2} = 1 \quad \boxed{\lambda = \pm\sqrt{2}}$

$2x^2 = \lambda \quad x^2 = \frac{\lambda}{2} \Rightarrow x^4 = \frac{\lambda^2}{4}$

$2y^2 = \lambda \quad y^2 = \frac{\lambda}{2} \Rightarrow y^4 = \frac{\lambda^2}{4}$

$x^2 = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} \quad y^2 = \frac{\sqrt{2}}{2}$

$\boxed{x = \pm\frac{1}{\sqrt[4]{2}} \quad y = \pm\frac{1}{\sqrt[4]{2}}}$

Max:

$\boxed{f\left(\pm\frac{1}{\sqrt[4]{2}}, \pm\frac{1}{\sqrt[4]{2}}\right) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}}$

14.8 (cont

11) $f(x,y,z) = 3x + 2y + 4z \quad g(x,y,z) = x^2 + 2y^2 + 6z^2 = 1$

$\nabla f = \langle 3, 2, 4 \rangle \quad \nabla g = \langle 2x, 4y, 12z \rangle$

$2x = 3\lambda \quad \lambda = \langle \frac{2}{3}x, 2y, 3z \rangle$

$4y = 2\lambda$

$12z = 4\lambda$

$x^2 + 2y^2 + 6z^2 = 1$

Solved in MATLAB:

$f_{min} = -3.7$

$f_{max} = 3.7$

13) $f(x,y,z) = xy + 2z \quad g(x,y,z) = x^2 + y^2 + z^2 = 36$

$\nabla f = \langle y, x, 2 \rangle \quad \nabla g = \langle 2x, 2y, 2z \rangle$

$2x = \lambda y \quad \lambda^2 = 4 \quad \lambda = \pm 2$

$2y = \lambda x \quad x = \pm y$

$2z = 2\lambda \quad z = \lambda = \pm 2$

$x^2 + y^2 + 4 = 36$

$x^2 + y^2 = 32$

$2y^2 = 32$

$y = \pm 4 \quad x = \pm 4$

Max

$f(4, 4, 2) = 20$

Min

$f(-4, +4, -2) = f(+4, -4, -2) = -20$

14.8 Cont

15) $f(x, y, z) = xy + xz \quad g(x, y, z) = x^2 + y^2 + z^2 = 4$

$\nabla f = \langle y+z,\ x,\ x \rangle \quad \nabla g = \langle 2x,\ 2y,\ 2z \rangle$

$y + z = 2x\lambda$

$x = 2y\lambda$

$x = 2z\lambda$

Solved Using MATLAB

$x = [0,\ 0,\ \sqrt{2},\ \sqrt{2},\ -\sqrt{2},\ -\sqrt{2}]$

$y = [-\sqrt{2},\ \sqrt{2},\ -1,\ 1,\ 1,\ -1]$

$z = [\sqrt{2},\ -\sqrt{2},\ -1,\ 1,\ 1,\ -1]$

Min Max Min Max

Min

$f(\sqrt{2}, -1, -1) = f(-\sqrt{2}, 1, 1) = -2\sqrt{2}$

Max

$f(\sqrt{2}, 1, 1) = f(-\sqrt{2}, -1, -1) = 2\sqrt{2}$