14.8

5. $\nabla f = \langle 2x, 2y \rangle$

$\nabla g = \langle 2, 3 \rangle$

$\cancel{2x}\ x = \lambda$

$2y = 3\lambda$

$2x+3y = \cancel{6\lambda} = 6.$
6.5λ

$\lambda = \frac{12}{13}$

$x = \frac{12}{13},\ y = \frac{18}{13}$

$f(\frac{12}{13}, \frac{18}{13}) = \frac{36}{13}$

$\therefore$ The minimum is $\frac{36}{13}$.

There's no maximum.

7. $\cancel{\nabla xy =}$

$\nabla f = \langle y, x \rangle$

$\nabla g = \langle 8x, 18y \rangle$

$y = 8x\lambda$

$x = 18y\lambda$

$4x^2+9y^2 = 1296y^2\lambda^2 + 576x^2\lambda^2 = 32$

$144(4x^2+9y^2)\lambda^2 = 32$

$\lambda^2 = \frac{1}{144}$

$\lambda = \pm\frac{1}{12}$

$\cancel{\lambda = \frac{1}{12}.}$

$\cancel{2x = 3y}$

$\cancel{4x^2 = 9y^2 = 16}$

$2x = \pm 3y$

$4x^2 = 9y^2 = 16$

$x = \pm 2,\ y = \pm\frac{4}{3}$

$f(2, \frac{4}{3}) = \frac{8}{3}$ is the maximum

$f(2, -\frac{4}{3}) = -\frac{8}{3}$ is the minimum

9. $\nabla f = \langle 2x, 2y \rangle$

$\nabla g = \langle 4x^3, 4y^3 \rangle.$

when both of them are not 0

$2x = 4x^3\lambda \quad \cancel{x} \quad 2x^2\lambda = 1$

$2y = 4y^3\lambda \quad\quad 2y^2\lambda = 1.$

$\cancel{4\lambda(x^3+y^3) = 2(x+y) =}$

$\cancel{\lambda(x^2+y^2)}$ $\quad 4x^4\lambda^2 = 1$

$4y^4\lambda^2 = 1.$

$4\lambda^2(x^4+y^4) = 2$

$\lambda^2 = \frac{1}{2}.$

$\lambda = \frac{\sqrt{2}}{2}$

$2\lambda(x^2+y^2) = 1.$

$x^2+y^2 = \sqrt{2}.$

when one of them is 0.

the other one is 1.

$x^2+y^2 = 1.$

$\therefore$ The maximum is $\sqrt{2}$.

The minimum is 1.

11. $\nabla f = \langle 3, 2, 4 \rangle$

$\nabla g = \langle 2x, 4y, 12z \rangle$

$3 = 2x\lambda$

$1 = 2y\lambda$

$1 = 3z\lambda$

$\lambda^2(x^2+2y^2+6z^2) = \frac{9}{4} + \frac{1}{2} + \frac{2}{3} = \frac{41}{12}$

$\lambda = \pm\sqrt{\frac{41}{12}}$

when $\lambda = \sqrt{\frac{41}{12}}$

$(x, y, z) = (0.81, 0.27, 0.18)$

$f(x,y,z) = 3.7$ is the maximum

when $\lambda = -\sqrt{\frac{41}{12}}$

$(x, y, z) = (-0.81, -0.27, -0.18)$

$f(x,y,z) = -3.7$ is the minimum

13. $\nabla f = \langle y, x, 2 \rangle$

$\nabla g = \langle 2x, 2y, 2z \rangle$

$y = 2x\lambda$

$x = 2y\lambda$

$1 = z\lambda$

$\cancel{\lambda^2(x^2+y^2+z^2) = \frac{y^2}{4} + \frac{x^2}{4} + 1}$

$4y\lambda^2 = y$

If x, y are not 0.

$\lambda = \pm\frac{1}{2}$.

$z = \pm 2$

$x = y = \pm\frac{\sqrt{36-4}}{2} = \pm 4$

$f(4,4,2) = 20$ is the maximum

$f(-4,-4,-2) = -20$ is the minimum

15. $\nabla f = \langle y+z, x, x \rangle$.

$\nabla g = \langle 2x, 2y, 2z \rangle$.

$y+z = 2x\lambda$

$x = 2y\lambda = 2z\lambda$

$\cancel{\lambda^2(x^2+y^2+z^2) =}$

$2x\lambda = \cancel{4\lambda^2(y+z)}$

$4\lambda^2 y = 4\lambda^2 z = y+z$.

$4\lambda^2(y+z) = 2(y+z)$

$\lambda = \frac{\sqrt{2}}{2}$, $\lambda = -\frac{\sqrt{2}}{2}$;

$x = \sqrt{2}y = \sqrt{2}z$, $x = -\sqrt{2}y = -\sqrt{2}z$

$\cancel{\sqrt{2}x = y+z = 2y}$

$y = z = \frac{4}{2+1+1} = \pm 1$.

$x = \pm\sqrt{2}$

$f(\sqrt{2},1,1) = 2\sqrt{2}$ is maximum

$f(-\sqrt{2},-1,-1) = -2\sqrt{2}$ is minimum