14.8

5)

$$f(x,y) = x^2 + y^2 \qquad , \; 2x + 3y = 6$$

$$\nabla f = \langle 2x, 2y \rangle \qquad \nabla g = \langle 2, 3 \rangle$$

$$2x = \lambda 2 \qquad\qquad 2y = \lambda 3$$
$$x = \lambda \qquad\qquad\qquad \lambda = \frac{2y}{3}$$

$$2\lambda + 2\lambda = 6 \qquad\qquad x = 3/2$$
$$2 \cancel{4} \lambda = \cancel{6} 3 \qquad\qquad y = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$$
$$\lambda = 3/2$$

7) $f(x,y) = xy \qquad\qquad 4x^2 + 9y^2 = 32$

$$\nabla f = \langle y, x \rangle \qquad \nabla g = \langle 8x, 18y \rangle$$

$$y = \lambda 8x \qquad\qquad x = \lambda 18 y$$
$$y = \lambda 8 (\lambda 18 y) \qquad\qquad = \lambda 18 \lambda 8 x$$
$$= 1y$$

$$\frac{y}{\lambda 8} = \lambda 18 y \qquad \text{\color{blue}{$y = 0$}}$$

$$\frac{1}{8 \times 18} = \lambda^2$$

$$\lambda = \frac{1}{3 \times 4} = \frac{1}{12}$$

9) $f(n, y) = n^2 + y^2$ $g = n^4 + y^4 = 1$

    $\triangledown f = \langle 2n, 2y \rangle$ $\triangledown g = \langle 4n^3, 4y^3 \rangle$

$$\cancel{2n} = \lambda \left(\cancel{4} n^{\cancel{3}} \right)^{\overset{2}{2}} \qquad 1 = \lambda (2y^2)$$

$$1 = \lambda n^2 \,{\color{blue}2} \qquad \qquad \left(\frac{1}{2\lambda} \right)^{\overset{2}{}} = y^4$$

$$\left(\frac{1}{2\lambda} \right)^2 = n^4$$

$$\frac{\cancel{2} 1}{2 \cancel{4} \lambda^2} = 1$$

$$\boxed{\dfrac{1}{\sqrt{2}} = \lambda}$$

$$x^2 = \dfrac{1}{2\lambda}$$

$$= \dfrac{\sqrt{2}}{2}$$

$$x^2 = \dfrac{1}{\sqrt{2}}$$

$$x = \dfrac{1}{4\sqrt{2}}$$

$$y^2 = \dfrac{\sqrt{2}}{2}$$

$$y = 4\dfrac{1}{\sqrt{2}}$$

11) $f(x,y,z) = 3x + 2y + 4z$

$\nabla f = \langle 3, 2, 4 \rangle$

$g = x^2 + 2y^2 + 6z^2 = 1$

$\nabla g = \langle 2x, 4y, 12z \rangle$

$$x = \frac{3}{2\lambda} \qquad y = \frac{1}{2\lambda} \qquad z = \frac{1}{3\lambda}$$

$$\frac{9}{4\lambda^2} + \frac{1}{2\lambda^2} + \frac{6^2}{3\mathfrak{A}\lambda^2} = 1$$

$$27 + 6 + 8 = 12\lambda^2$$

$$\lambda = 1.8\Delta 8$$

$$x = 1.103$$

$$y = 2.719$$

$$z = 0.18D3$$

B) $f(x,y,z) = xy + 2z$

$\nabla f = \langle y, x, 2 \rangle$ $\langle 2x, 2y, 2z \rangle$

$2x = \lambda y$ $x = 2\lambda y$ $2 = \lambda 2z$

$$z = \frac{1}{\lambda}$$

$$2 + \frac{1}{\lambda^2} = 36$$

$$\frac{1}{\lambda^2} = 34$$

$$\frac{1}{\sqrt{34}} = \lambda$$

(5)

$$\nabla f \quad \langle y+z, \; x, \; x \rangle$$

$$\langle 2x, 2y, 2z \rangle$$

$$x = \frac{y+z}{2\lambda} \qquad x = \lambda 2y \qquad\qquad x = \lambda 2z$$

$$y = z$$

$$x = \frac{2y}{2\lambda}$$

$$\frac{x}{\lambda^2 2}$$

$$x = 1$$

$$1 + 2y^2 = 4$$

$$1 + 2\left(\frac{1}{4\lambda^2}\right) = 4$$

$$\frac{1}{2\lambda^2} = 3$$

$$\frac{1}{\lambda^2} = 6$$

$$\lambda = \frac{1}{\sqrt{6}}$$