

homework 6

14.8

5)  $f(x,y) = x^2 + y^2$      $g(x,y) = 2x + 3y = 6$   
 $\nabla f = (2x, 2y)$      $\nabla g = (2, 3)$

$2x = 2\lambda$     ~~or~~  $x = \lambda$     ~~(2(2) + 3(3/2)) = 6~~  
 $2y = 3\lambda$      $y = \frac{3\lambda}{2}$

$\frac{4\lambda}{2} + 3\left(\frac{3\lambda}{2}\right) = 6 \Rightarrow 4\lambda + 9\lambda = 12 \Rightarrow 13\lambda = 12 \Rightarrow \lambda = \frac{12}{13}$

$x = \lambda \rightarrow x = \frac{12}{13}$   
 $y = \frac{3\lambda}{2} \rightarrow y = \frac{18}{13}$  }  $\rightarrow \left(\frac{12}{13}\right)^2 + \left(\frac{18}{13}\right)^2 = \frac{18}{13} + \frac{18}{13} = \frac{36}{13}$  min  
 no max

7)  $f(x,y) = xy$      $g(x,y) = 4x^2 + 9y^2 = 32$   
 $\nabla f = (y, x)$      $\nabla g = (8x, 18y)$

~~$y = 8x^2 \rightarrow 4(8x^2)^2 + 9(8x^2)^2 = 32$~~   
 ~~$x = 18y \rightarrow 4(18y)^2 + 9(18y)^2 = 32$~~   
 ~~$\lambda = \frac{y}{8x}$~~   
 ~~$\lambda = \frac{x}{18y}$~~

$\frac{y}{8x} = \frac{x}{18y} \Rightarrow 18y^2 = 8x^2$      $\frac{18y^2}{8} + y^2 = 32$   
 $x = \sqrt{\frac{18}{8}} y$      $\frac{18y^2}{8} + \frac{8y^2}{8} = 32$

~~$4\left(\frac{18x}{8y}\right)^2 + 9x^2 = 32$~~   
 ~~$9\left(\frac{8y}{18x}\right)^2 + 9y^2 = 32$~~   
 $\frac{8x^2}{18} + x^2 = 32$      $36y^2 = 256$   
 $26x^2 = 576$      $y^2 = \frac{64}{9}$   
 $x^2 = \frac{576}{26} + 1$

$f\left(\pm 1, \frac{8}{3}\right) = \boxed{\pm \frac{8}{3}}$

9.)  $f(x,y) = x^2 + y^2$       $g = x^4 + y^4 - 1$   
 $\nabla f = \langle 2x, 2y \rangle$       $\nabla g = \langle 4x^3, 4y^3 \rangle$   
 $2x = 4x^3 \lambda$

$2y = 4y^3 \lambda$       $x = 2x^3 \lambda$       $(2x^3 \lambda)^4 + (2y^3 \lambda)^4 = 1$   
 $y = 2y^3 \lambda$       $16x^{12} \lambda^4 + 16y^{12} \lambda^4 = 1$   
 $x^{12} \lambda^4 + y^{12} \lambda^4 = 1/16$   
 $2x^{12} \lambda^4 = 1/16$

$\lambda = \frac{x \cdot y}{4x^3} = \lambda = \frac{y}{4x^2}$   
~~HAHA~~  $\lambda = \frac{\sqrt{2}}{4}$       $\lambda = \frac{1}{4}$   
max     min

$x^{12} \lambda^4 = \frac{1}{32}$   
 $x^{12} \frac{1}{x^2} = \frac{1}{32}$   
 $x^{10} = \frac{1}{32}$

AM  $x, y = \frac{1}{\sqrt{2}}$

11.)  $f(x,y,z) = x^2 + y^2 + z^2$       $g = x^2 + 2y^2 + 10z^2 - 1$   
 $\nabla f = \langle 2x, 2y, 2z \rangle$       $\nabla g = \langle 2x, 4y, 10z \rangle$

$3 = 2x \lambda$       $x = \frac{3}{2\lambda}$       $x = \frac{3}{2\lambda}$   
 $2 = 4y \lambda$       $y = \frac{1}{2\lambda}$       $y = \frac{1}{2\lambda}$   
 $4 = 10z \lambda$       $z = \frac{2}{5\lambda}$       $z = \frac{2}{5\lambda}$

$(\frac{3}{2\lambda})^2 + 2(\frac{1}{2\lambda})^2 + 10(\frac{2}{5\lambda})^2 = 1$   
 $\frac{9}{4\lambda^2} + \frac{1}{2\lambda^2} + \frac{8}{\lambda^2} = 1$   
 $\frac{21}{12\lambda^2} + \frac{6}{12\lambda^2} + \frac{64}{12\lambda^2} = 1$

~~HAHA~~  $\frac{91}{12\lambda^2} = 1$       $12\lambda^2 = 91$       $\lambda = \sqrt{\frac{91}{12}}$

$x = 0.843$   
 $y = 0.26$   
 $z = 0.187$

$\frac{38}{12\lambda^2} = 1$       $12\lambda^2 = 38$       $\lambda = \sqrt{\frac{38}{12}}$

± 38

$$13.) f(x, y, z) = xy + 2z$$

$$\nabla f = \langle y, x, 2 \rangle$$

$$g: x^2 + y^2 + z^2 = 30$$

$$\nabla g = \langle 2x, 2y, 2z \rangle$$

$$y = 2\lambda$$

$$x = 2\lambda$$

$$2z = 2\lambda$$

$$\lambda = 1$$

$$x = 2, y = 2$$

$$4 + 4 + z^2 = 30$$

$$z = \sqrt{22}$$

$$xy + 2z = (2)(2) + 2(\sqrt{22}) = 18.5$$

$$x = \pm 4, y = \pm 4, z = 2$$

$$\boxed{\text{max} = 20, \text{min} = -20}$$

$$15.) f(x, y, z) = xy + xz$$

$$\nabla f = \langle y+z, x, x \rangle$$

$$g: x^2 + y^2 + z^2 = 4$$

$$\nabla g = \langle 2x, 2y, 2z \rangle$$

$$x = 2\lambda$$

$$x = 2\lambda$$

$$y+z = 2\lambda$$

$$y = \sqrt{2\lambda} \quad y = 2\lambda - z$$

$$z = 2\lambda - y$$

$$(2\lambda)^2 + (2\lambda - z)^2 + (2\lambda - y)^2 = 4$$

$$4\lambda^2 + 4\lambda^2 - 2\lambda z + z^2 + z^2 = 4$$

$$8\lambda^2 - 2\lambda z + 2z^2 = 4$$

$$4\lambda^2 - \lambda z + z^2 = 2$$

$$xy = \sqrt{2}$$

$$\sqrt{2} + \sqrt{2} = \pm 2\sqrt{2}$$

$$xz = \sqrt{2}$$

$$\boxed{\text{max} = 2\sqrt{2}$$

$$\text{min} = -2\sqrt{2}}$$