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5. $f(x, y) = x^2 + y^2$, $2x + 3y = 6$

$f_x = 2x$ $g_x = 2$

$f_y = 2y$ $g_y = 3$

$x = \lambda$

$y = \frac{3}{2}\lambda$

$\nabla f = \langle 2x, 2y \rangle$

$2\lambda + \frac{9}{2}\lambda = 6$

$\nabla g = \langle 2, 3 \rangle$

$\frac{13}{2}\lambda = 6$

$\nabla f = \lambda \nabla g$

$\lambda = \frac{12}{13}$

$\langle 2x, 2y \rangle = \lambda \langle 2, 3 \rangle$

$x = \frac{12}{13}$

$2x = 2\lambda$ $2x + 3y = 6$

$y = \frac{36}{26}$

$2y = 3\lambda$ $4xy = 6\lambda^2$

$f\left(\frac{12}{13}, \frac{36}{26}\right) = \left(\frac{12}{13}\right)^2 + \left(\frac{36}{26}\right)^2$
 $= \frac{36}{13}$

7.

$f(x, y) = xy$, $4x^2 + 9y^2 = 32$

$f_x = y$ $g_x = 8x$

$f_y = x$ $g_y = 18y$

$144\lambda = x$

$8 = y$

$\frac{1}{32} = \lambda$

$12 = x$

$\nabla f = \langle y, x \rangle$

$\nabla g = \langle 8x, 18y \rangle$

$f\left(12, \frac{1}{12}\right) = 1$

$\nabla f = \lambda \nabla g$

$y = 8x\lambda$ $4x^2 + 9y^2 = 32$

$x = 18y\lambda$ $xy = 144y\lambda$

9.

$$f(x, y) = x^2 + y^2, \quad x^4 + y^4 = 1$$

$$f_x = 2x \quad g_x = 4x^3$$

$$f_y = 2y \quad g_y = 4y^3$$

$$\left(\frac{1}{4x}\right)^4 + \left(\frac{1}{4y}\right)^4 = 1$$

$$\langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle$$

$$2x = 4x^3 \lambda \quad xy = 16x^2 y^2 \lambda^2$$

$$2y = 4y^3 \lambda \quad \frac{1}{4xy} = \lambda$$

$$\frac{1}{256x^4} + \frac{1}{256y^4} = 1$$

$$\frac{2}{256x} = 1$$

$$x = 4x^3 \lambda \quad x = \frac{1}{4\lambda} = 8\sqrt{2}$$

$$y = 4y^3 \lambda \quad y = \frac{1}{4\lambda} = 8\sqrt{2}$$

$$2 = 256\lambda^4$$

$$\frac{1}{128} = \lambda^4$$

$$\frac{1}{\sqrt{128}} = \lambda$$

$$f(8\sqrt{2}, 8\sqrt{2}) = (8\sqrt{2})^2 + (8\sqrt{2})^2 = 256$$

11.

$$f(x, y, z) = 3x + 2y + 4z, \quad x^2 + 2y^2 + 6z^2 = 1$$

$$f_x = 3 \quad g_x = 2x$$

$$f_y = 2 \quad g_y = 4y$$

$$f_z = 4 \quad g_z = 12z$$

$$\Rightarrow (3)\left(\frac{3\sqrt[3]{6^2}}{2}\right) + (2)\left(\frac{\sqrt[3]{6^2}}{2}\right) + (4)\left(\frac{\sqrt[3]{6^2}}{3}\right)$$

$$= \frac{41\sqrt[3]{6}}{6}$$

$$\langle 3, 2, 4 \rangle = \lambda \langle 2x, 4y, 12z \rangle$$

$$3 = 2x\lambda \quad 24 = 96xyz\lambda^3 \Rightarrow 24 = 96\left(\frac{3}{6\lambda}\right)\left(\frac{2}{6\lambda}\right)\left(\frac{4}{6\lambda}\right)\lambda^3$$

$$2 = 4y\lambda \quad x = \frac{3}{2\lambda} \quad z = \frac{1}{3\lambda} \quad \frac{1}{36} = \lambda^3 \quad x = \frac{3 \cdot \sqrt[3]{6^2}}{2}$$

$$4 = 12z\lambda \quad y = \frac{1}{2\lambda} \quad \frac{1}{\sqrt[3]{36}} = \lambda \quad z = \frac{\sqrt[3]{6^2}}{3}$$

15.

$$f(x, y, z) = xy + xz, \quad x^2 + y^2 + z^2 = 4$$

$$\begin{aligned}
 f_x &= 2 & g_x &= 2x \\
 f_y &= 1 & g_y &= 2y \\
 f_z &= 1 & g_z &= 2z
 \end{aligned}$$

$$\begin{aligned}
 f(2, 1, 1) &= (2)(1) + (2)(1) \\
 &= 4
 \end{aligned}$$

$$\langle 2, 1, 1 \rangle = \lambda \langle 2x, 2y, 2z \rangle$$

$$2 = 2x\lambda \quad 2 = 8xyz\lambda^3 \Rightarrow 2 = \frac{8}{x}\lambda^3$$

$$1 = 2y\lambda \quad x = \frac{2}{\lambda} = 2 \quad \Rightarrow 2 = 8\lambda^2$$

$$1 = 2z\lambda \quad y = \frac{1}{2\lambda} = 1 \quad \Rightarrow \frac{1}{4} = \lambda^2$$

$$z = \frac{1}{2\lambda} = 1 \quad \frac{1}{2} = \lambda$$

