14.8

5. $f(x,y) = x^2 + y^2 \;\; g(x,y) = 2x + 3y \; g(x,y) = 6$

$$\nabla f(x,y) = (2x, 2y) \;\; \nabla g(x,y) = (2,3) \;\rightarrow \begin{cases} 2x = 2\mu \\ 2y = 3\mu \\ 2x + 3y = 6 \end{cases} \rightarrow \begin{cases} \mu = \dfrac{12}{13} \\ x = \dfrac{12}{13} \\ y = \dfrac{18}{13} \end{cases}$$

$$so\ the\ minimum\ of\ f(x,y)\ is\ f\left(\frac{12}{13}.\frac{18}{13}\right) = \frac{36}{13}\ and\ the\ maximum\ is + \infty$$

7. $f(x,y) = xy \; g(x,y) = 4x^2 + 9y^2 \; g(x,y) = 32$

$$\nabla f(x,y) = (y,x) \; \nabla g(x,y) = (8x, 18y) \rightarrow \begin{cases} \mu y = 8x \\ \mu x = 18y \\ 4x^2 + 9y^2 = 32 \end{cases} \rightarrow \begin{cases} \mu = \pm 12 \\ x = \pm 2 \\ y = \pm \dfrac{4}{3} \end{cases}$$

$$So, the\ max\ of\ f(x,y)\ is\ \frac{8}{3}\ and\ the\ min\ is\ -\frac{8}{3}.$$

9. $f(x,y) = x^2 + y^2 \; g(x,y) = x^4 + y^4 \; g(x,y) = 1$

$$\nabla f(x,y) = (2x, 2y) \; \nabla g(x,y) = (4x^3, 4y^3) \rightarrow \begin{cases} 2\mu x = 4x^3 \\ 2\mu y = 4y^3 \\ x^4 + y^4 = 1 \end{cases} \rightarrow \begin{cases} \mu = \sqrt{2} \\ x = \pm 2^{-\frac{1}{4}} \\ y = \pm 2^{-\frac{1}{4}} \end{cases}$$

$$so, the\ min\ of\ f(x,y)\ is\ 1\ and\ the\ max\ of\ f(x,y)\ is\ \sqrt{2}$$

11. $f(x,y,z) = 3x + 2y + 4z \; g(x,y,z) = x^2 + 2y^2 + 6z^2 \; g(x,y,z) = 1$

$$\nabla f(x,y,z) = (3,2,4) \; \nabla g(x,y,z) = (2x, 4y, 12z) \rightarrow \begin{cases} 3\mu = 2x \\ 2\mu = 4y \\ 4\mu = 12z \\ x^2 + 2y^2 + 6z^2 = 1 \end{cases}$$

$$\begin{cases} \mu = \pm\sqrt{\frac{12}{41}} \\ x = \pm\frac{3}{2} * \sqrt{\frac{12}{41}} \\ y = \pm\frac{1}{2} * \sqrt{\frac{12}{41}} \\ z = \pm\frac{1}{3} * \sqrt{\frac{12}{41}} \end{cases}, so\ the \min of\ f(x,y,z)\ is - 3.7\ and\ the \max is\ 3.7$$

13. $f(x,y,z) = xy + 2z \; g(x,y,z) = x^2 + y^2 + z^2 \; g(x,y,z) = 36$

$$\nabla f(x,y,z) = (y,x,2) \; \nabla g(x,y,z) = (2x, 2y, 2z) \rightarrow \begin{cases} \mu y = 2x \\ \mu x = 2y \\ 2\mu = 2z \\ x^2 + y^2 + z^2 = 36 \end{cases} , \mu = \pm 2$$

$$so\ the\ maxima\ of\ f(x,y,z)\ is\ 20\ and\ minima\ is - 20$$

15. $f(x,y,z) = xy + xz$ $g(x,y,z) = x^2 + y^2 + z^2$ $g(x,y,z) = 4$

$\nabla f(x,y,z) = (y+z, x, x)$ $\nabla g(x,y,z) = (2x, 2y, 2z)$

$$\begin{cases} 2\mu x = y + z \\ 2\mu y = x \\ 2\mu z = x \\ x^2 + y^2 + z^2 = 4 \end{cases} \rightarrow \mu = \pm\frac{\sqrt{2}}{2}$$

so the minima of $f(x,y,z)$ *is* $-2\sqrt{2}$ *and the maxima of* $f(x,y,z)$ *is* $2\sqrt{2}$