10/17/20 14.8 Lagrange Multipliers: Optimizing with a Constraint

#5, 7, 9, 11, 13, 15.

5). $f(x,y) = x^2 + y^2$ $\quad 2x + 3y = 6$

$\nabla f = \langle 2x, 2y \rangle$ $\quad \nabla g = \langle 2, 3 \rangle$.

$2x = \lambda(2)$ $\quad 2y = 3\lambda$

$\lambda = x$ $\quad 2y = 3x$

$\frac{2y}{3} = \lambda$

$\lambda = x = \frac{2y}{3}$.

$2x + 3(\frac{3}{2}x) - 6 = 0$.

$\frac{13}{2}x = 6$

$x = \frac{12}{13}$.

$y = \frac{18}{3}$

$f(\frac{12}{13}, \frac{18}{13}) = \frac{36}{13}$.

7) $f(x,y) = xy$ $\quad 4x^2 + 9y^2 = 32$.

$\nabla f = \langle y, x \rangle$ $\quad \nabla g = \langle 8x, 18y \rangle$.

$y = 8x \cdot \lambda$ $\quad x = 18y \cdot \lambda$

$\frac{y}{8x} = \frac{x}{18y} = \lambda$.

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$4x^2 - 9y^2 - 32 = 0$

$8x^2 = 32$

$x^2 = 4$

$f(2, \frac{4}{3})$ $x = \pm 2$ $= \frac{8}{3}$

$f(-2, \frac{-4}{3})$ $= -\frac{8}{3}$

9) $f(x,y) = x^2 + y^2$ $x^4 + y^4 = 1$

$\nabla f = \langle 2x, 2y \rangle$ $\nabla g = \langle 4x^3, 4y^3 \rangle$

$2x = 4x^3(\lambda)$

$\lambda = \frac{1}{2x^2} = \frac{1}{2y^2}$

$x^4 + y^4 - 1 = 0$

$x^4 + x^4 - 1 = 0$

$2x^4 = 1$

$x = \frac{1}{2^{\frac{1}{4}}}$, $y = \frac{1}{2^{\frac{1}{4}}}$

$f\left(\frac{1}{2^{\frac{1}{4}}}, \frac{1}{2^{\frac{1}{4}}}\right) = \sqrt{2}$

11) $f(x,y,z) = 3x + 2y + 4z$, $x^2 + 2y^2 + 6z^2 = 1$.
$\nabla f = \langle 3, 2, 4 \rangle$, $\nabla g = \langle 2x, 4y, 12z \rangle$

$3 = 2x(\lambda)$, $2 = 4y(\lambda)$, $4 = 12z(\lambda)$

$\lambda = \frac{3}{2x} = \frac{1}{2y} = \frac{1}{3z}$

$x^2 + 2y^2 + 6z^2 - 1 = 0$

$x^2 + \left(\frac{x}{3}\right)^2 + \left(\frac{2x}{9}\right)^2 - 1 = 0$.

$\frac{123}{81}x^2 - 1 = 0$.

$x^2 = \pm\sqrt{\frac{81}{123}} = \frac{9}{\pm\sqrt{123}}$

$f(xyz) = \frac{41}{\sqrt{123}}$

13) $f(x,y,z) = xy + 2z$, $x^2 + y^2 + z^2 = 36$.
$\nabla f = \langle y, x, 2 \rangle$. $\nabla g = \langle 2x, 2y, 2z \rangle$.

$y = \lambda(2x)$ $x = \lambda(2y)$, $2 = (2z)\lambda$.

$y = 2\lambda(2\lambda y)$

$y(1 - 4\lambda^2) = 0$.

$x^2 \quad + x^2 \quad + 4 - 36 = 0$

$2x^2 \quad = 32$

$x = \pm 4$

$(4, 4, 2)$

$f(4, 4, 2) = 20.$

$f(4, -4, 2) = -20$

$f(4, 4, -2) = 12$

$f(4, -4, 2) = -12$

$f(0, 0, 6) = 12$

15) $f(x, y, z) = xy + xz$ $\qquad x^2 + y^2 + z^2 = 4$

$\nabla f = \langle y + z, x, x \rangle$ $\qquad \nabla g = \langle 2x, 2y, 2z \rangle$

$y + z = \lambda(2x) \qquad x = 2y(\lambda) \qquad x = 2z(\lambda)$

$x^2 + y^2 + z^2 - 4 = 0$

$2y^2 - 4 = \quad y = \pm\sqrt{2}$

$2\lambda \quad = \frac{y+z}{x}, \quad \frac{2\lambda}{x} = y = \quad z$

$2\lambda = \frac{y+z}{x} = \frac{x}{y} \qquad y = x$

$\frac{2y}{x} = \frac{x}{y} \qquad x^2 = 2y^2$

$x^2+y^2+z^2-4=0$

$2y^2+y^2+y^2-4=0$

$4y^2=4$

$y=\pm 1$

Critical Points $\rightarrow$ $(\pm\sqrt{2}, 1, 1)$

$(\pm 2, -1, -1)$,

$(0, \pm\sqrt{2}, \pm\sqrt{2})$

$f(\pm\sqrt{2}, 1, 1) = 2\sqrt{2}$ Max

$f(\pm\sqrt{2}, -1, -1) = -2\sqrt{2}$ Min

$f(0, \pm\sqrt{2}, \pm\sqrt{2}) = 0$