

$$5. f(x, y) = x^2 + y^2, \quad 2x + 3y = 6$$

$$\nabla f = \langle 2x, 2y \rangle, \quad \nabla g = \langle 2, 3 \rangle$$

$$\nabla f = \lambda \nabla g$$

$$2x = 2\lambda, \quad 2y = 3\lambda, \quad 2x + 3y = 6$$

$$x = \lambda, \quad y = \frac{3}{2}\lambda \quad 2\lambda + \frac{9}{2}\lambda = 6$$

$$\frac{13}{2}\lambda = 6$$

$$\lambda = \frac{12}{13}$$

$$x = \frac{12}{13}, \quad y = \frac{18}{13}$$

$$f\left(\frac{12}{13}, \frac{18}{13}\right) = \frac{144 + 324}{169} = \frac{468}{169} = \frac{36}{13}$$

$$7. f(x, y) = xy \quad 4x^2 + 9y^2 = 32$$

$$\nabla f = \langle y, x \rangle \quad \nabla g = \langle 8x, 18y \rangle$$

$$\nabla f = \lambda \nabla g$$

~~$$y = 8\lambda x, \quad x = 18\lambda y, \quad 4x^2 + 9y^2 = 32$$~~

$$y = 144\lambda^2 y, \quad x = 144\lambda^2 x$$

$$144\lambda^2 = 1$$

$$\lambda = \frac{1}{12}$$

$$x = \frac{3}{2}y, \quad y = \frac{2}{3}x, \quad 4x^2 + 4x^2 = 32$$

$$8x^2 = 32$$

$$x = 2 \text{ or } -2$$

$$y = \frac{4}{3} \text{ or } -\frac{4}{3}$$

$$\max f\left(2, \frac{4}{3}\right) = \frac{8}{3} \quad \min f\left(-2, -\frac{4}{3}\right) = -\frac{8}{3}$$

$$9. f(x, y) = x^2 + y^2, \quad x^4 + y^4 = 1$$

$$\nabla f = \langle 2x, 2y \rangle \quad \nabla g = \langle 4x^3, 4y^3 \rangle$$

$$\nabla f = \lambda \nabla g$$

$$2x = 4\lambda x^3, \quad 2y = 4\lambda y^3, \quad x^4 + y^4 = 1$$

$$\lambda = \frac{1}{2}$$

~~$$x = x, \quad y = y$$~~



$$11. f(x, y, z) = 3x + 2y + 4z, \quad x^2 + 2y^2 + 6z^2 = 1$$

$$\nabla f = \langle 3, 2, 4 \rangle, \quad \nabla g = \langle 2x, 4y, 12z \rangle$$

$$\lambda \nabla f = \nabla g$$

$$\lambda 3 = 2x, \quad \lambda 2 = 4y, \quad \lambda 4 = 12z, \quad x^2 + 2y^2 + 6z^2 = 1$$

$$x = \frac{3}{2}\lambda, \quad y = \frac{1}{2}\lambda, \quad z = \frac{1}{3}\lambda, \quad \frac{9}{4}\lambda^2 + \frac{1}{2}\lambda^2 + \frac{2}{3}\lambda^2 = 1$$

$$\frac{41}{12}\lambda^2 = 1$$

$$\lambda^2 = \frac{12}{41}$$

$$\lambda = \pm \sqrt{\frac{12}{41}}$$

$$13. f(x, y, z) = xy + 2z, \quad x^2 + y^2 + z^2 = 36$$

$$\nabla f = \langle y, x, 2 \rangle, \quad \nabla g = \langle 2x, 2y, 2z \rangle$$

$$\lambda \nabla f = \nabla g$$

$$\lambda y = 2x, \quad \lambda x = 2y, \quad \lambda 2 = 2z, \quad x^2 + y^2 + z^2 = 36$$

$$\frac{\lambda y}{2} = x, \quad \frac{\lambda x}{2} = y, \quad \lambda = z$$

$$\lambda = 2, \quad x = \pm 4, \quad y = \pm 4, \quad z = \pm 2$$

$$f(4, 4, 2) = 20, \quad f(4, -4, -2) = -20$$

$$15. f(x, y, z) = xy + xz, \quad x^2 + y^2 + z^2 = 4$$

$$\nabla f = \langle y+z, x, x \rangle, \quad \nabla g = \langle 2x, 2y, 2z \rangle$$

$$\lambda \nabla f = \nabla g$$

$$\lambda(y+z) = 2x, \quad \lambda x = 2y, \quad \lambda x = 2z, \quad x^2 + y^2 + z^2 = 4$$

$$2y = 2z$$

$$y = z$$

$$\lambda 2y = 2x$$

$$\lambda y = x$$

$$x^2 + 2y^2 = 4$$

$$\lambda^2 y^2 + 2y^2 = 4$$

$$(\lambda^2 + 2)y^2 = 4$$

