rA

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Calc 3
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14.8: 5, 7, 9, 11, 13, 15

#5 $f(x,y) = 2x + 3y$, $x^2 + y^2 = 4.$

$\Delta f = \langle 2, 3 \rangle \quad \Delta g = \langle 2x, 2y \rangle$

$\langle 2,3 \rangle = \lambda \langle 2x + 2y \rangle$

$2 = \lambda 2x \quad 3 = \lambda 2y \quad x^2 + y^2 = 4.$

$6 = \lambda 2x \cdot \lambda 2y$

$6 = \lambda 2x \cdot 3 \qquad 6 = 2 \cdot \lambda 2y$

$2 = \lambda 2x \qquad 3 = \lambda 2y$

$1 = \lambda x$

$\underline{y = 3/2\lambda}$

$\underline{x = 1/\lambda}$

$(1/\lambda)^2 + (3/2\lambda)^2 = 4$

$1/\lambda^2 + 9/4\lambda^2 = 4$

$4/4\lambda^2 + 9/4\lambda^2 = 4$

$\frac{13}{4\lambda^2} = \frac{4}{1} \qquad \frac{13}{16} = \lambda^2$

$13 = 16\lambda^2$

$\underline{\lambda = \pm\sqrt{13/16}}$

$x = \frac{1}{\sqrt{13/16}}, \ -\frac{1}{\sqrt{13/16}}$

$y = 3/2\sqrt{13/16}, \ -3/2\sqrt{13/16}$

$P_1: (1/\sqrt{13/16}, \ -1/\sqrt{13/16})$

$P_2 = (3/2\sqrt{13/16}, \ -3/2\sqrt{13/16})$

$P_1: 2\left(\frac{1}{\sqrt{13/16}}\right) + 3\left(\frac{3}{2\sqrt{13/16}}\right) = \frac{4}{2\sqrt{13/16}} + \frac{9}{2\sqrt{13/16}} = \boxed{\frac{13}{2\sqrt{13/16}}} \longleftarrow$ Max

$P_2: 2\left(-\frac{1}{\sqrt{13/16}}\right) + 3\left(-\frac{3}{2\sqrt{13/16}}\right) = -\frac{4}{2\sqrt{13/16}} - \frac{9}{2\sqrt{13/16}} = \boxed{-\frac{13}{2\sqrt{13/16}}} \longleftarrow$ Min

#7 $f(x,y) = xy$, $4x^2 + 9y^2 = 32$

$\Delta f = \langle y, x \rangle \quad \Delta g = \langle 8x, 18y \rangle$

$\langle y, x \rangle = \lambda \langle 8x, 18y \rangle$

$y = \lambda 8x \quad x = \lambda 18y \quad 4x^2 + 9y^2 = 32$

$xy = \lambda 8x \cdot \lambda 18y$

$\cancel{xy} = \cancel{y} \cdot \lambda 18 \cdot \lambda 8 \cancel{x}$

$1 = 144\lambda^2$

$1/144 = \lambda^2$

$\lambda = \pm\sqrt{1/144} = \pm\frac{1}{12}$

$y_1 = \frac{1}{12} \cdot 8x = 2x/3$

$y_2 = -\frac{1}{12} \cdot 8x = -2x/13$

$x_1 = \frac{1}{12} \cdot 18y = 3y/2$

$x_2 - \frac{1}{12} \cdot 18y = -3y/2$

Substitute

$4(\pm 3y/2)^2 + 9y^2 = 32$

$4(9y^2/4) + 9y^2 = 32$

$9y^2 + 9y^2 = 32$

$18y^2 = 32$

$y^2 = 16/9$

$y = \pm\sqrt{16/9}$

$y = \pm\frac{4}{3}$

$x = \pm 3(\pm 4/3)/2$

$x = \pm 2$

$\frac{4}{3} \cdot 2$ or $\left(-\frac{4}{3}\right)(-2)$

$\frac{8}{3}$ and $\frac{8}{3}$

$\boxed{\frac{8}{3} \text{ is the max}}$

#9 $f(x,y) = x^2+y^2 \qquad x^4+y^4=1$

$\Delta f = \langle 2x, 2y\rangle \quad \Delta g = \langle 4x^3, 4y^3\rangle$

$\langle 2x, 2y\rangle = \lambda \langle 4x^3, 4y^3\rangle$

$2x = 4\lambda x^3 \quad 2y = 4\lambda y^3$
$x = 2\lambda x^3 \quad y = 2\lambda y^3$

$xy = 2\lambda x^3 \cdot 2\lambda y^3$

$1 = 4\lambda^2 x^2 y^2$

$\sqrt{\frac{1}{4\lambda^2 x^2}} = \sqrt{y^2} \quad \Big| \quad \sqrt{\frac{1}{4\lambda^2 y^2}} = \sqrt{x^2}$

$\frac{1}{2\lambda x} = y \quad \Big| \quad \frac{1}{2\lambda y} = x$

$\lambda = \frac{1}{2\lambda x} = \frac{1}{2\lambda y}$

$x^4 + x^4 = 1 \qquad y^4+y^4=1$
$2x^4 = 1 \qquad 2y^4 = 1$
$x^4 = 1/2 \qquad y^4 = 1/2$
$x = \pm\sqrt[4]{1/2} \qquad y = \sqrt[4]{1/2}$

$x^2+y^2 \longrightarrow (\pm 1/2^{1/4})^2 + (\pm 1/2^{1/4})^2 =$

$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \boxed{\frac{2}{\sqrt{2}}}$

#11 $f(x,y,z) = 3x+2y+4z, \quad x^2+2y^2+6z^2=1$

$\Delta f = \langle 3, 2, 4\rangle \quad \Delta g = \langle 2x, 4y, 12z\rangle$

$\langle 3,2,4\rangle = \lambda\langle 2x, 4y, 12z\rangle$

$3 = \lambda 2x \quad 2 = \lambda 4y \quad 4 = \lambda 12z$

$24 = \lambda 2x \cdot \lambda 4y \cdot \lambda 12 \cdot z$

$24 = \lambda 2x \cdot 2 \cdot 4 \qquad 24 = 3 \cdot \lambda 4y \cdot 4 \qquad 24 = 3 \cdot 2 \cdot \lambda 12z$
$3 = \lambda 2x \qquad 2 = \lambda 4y \qquad 4 = \lambda 12z$
$x = 3/\lambda 2 \qquad y = 1/\lambda 2 \qquad z = 1/\lambda 3$

$(3/\lambda 2)^2 + 2(1/\lambda 2)^2 + 6(1/\lambda 3)^2 = 1$

$9/\lambda^2 4 + 2/\lambda^2 4 + 6/\lambda^2 9 = 1$

$\frac{1}{\lambda^2}\left(\frac{9}{4} + \frac{1}{2} + \frac{6}{9}\right) = 1$

$\frac{41}{12} = \lambda^2$

$\lambda = \pm\sqrt{\frac{41}{12}}$

$x = \frac{3}{2\sqrt{\frac{41}{12}}} \qquad y = \frac{1}{2\sqrt{\frac{41}{12}}} \qquad z = \frac{1}{3\sqrt{\frac{41}{12}}}$

$3\left(\frac{3}{2\sqrt{\frac{41}{12}}}\right) + 2\left(\frac{1}{2\sqrt{\frac{41}{12}}}\right) + 4\left(\frac{1}{3\sqrt{\frac{41}{12}}}\right) = \boxed{3.69685}$

#13 $f(x,y,z) = xy + 2z \qquad x^2+y^2+z^2 = 36$

$f\Delta = \langle y, x, 2\rangle \quad g\Delta = \langle 2x, 2y, 2z\rangle$

$\langle y, x, 2\rangle = \lambda\langle 2x, 2y, 2z\rangle$

$y = \lambda 2x \quad x = \lambda 2y \quad 2 = \lambda 2z \rightarrow z = \frac{1}{\lambda}$

$\cancel{y} = \lambda 2\, \lambda 2 \cancel{y} \qquad y = x \quad y = -x \quad x = -y$
$1 = 4\lambda^2$
$\pm\frac{1}{2} = \lambda$

$z = \pm 2$

$x^2 + x^2 + 4 = 36$
$2x^2 = 32$
$x^2 = 16$
$x = \pm 4$

$y^2 + y^2 + 4 = 32$
$y = \pm 4$

$P_1\ (4,4,2) \qquad P_2\ (-4,-4,-2)$

1) $4 \cdot 4 + 2(2) = 20$
2) $(-4)(-4) + 2(-2) = 12$

$\boxed{\begin{array}{c}\text{Max : } 20 \\ \text{Min : } 12\end{array}}$

#15 $f(x,y,z)$ $xy + xz$ $x^2 + y^2 + z^2 = 4$

$\Delta f = (y+z, x, x)$ $\Delta g = (2x, 2y, 2z)$

$(y+z, x, x) = \lambda(2x, 2y, 2z)$

$y+z = \lambda 2x$ $x = \lambda 2y = \lambda 2z$

$\frac{x}{\lambda 2} + \frac{x}{\lambda 2} = \lambda 2x$

$\frac{2x}{\lambda \cancel{2}} = \lambda 2x$

$\frac{x}{\lambda} = \frac{\lambda 2x}{1}$

$\lambda^2 2 \cancel{x} = \cancel{x}$

$\lambda^2 = \frac{1}{2}$

$\lambda = \pm\sqrt{\frac{1}{2}}$

$x = 2\sqrt{1/2}\, y = 2\sqrt{\frac{1}{2}}\, z$

$x = y = z$

$x^2 + x^2 + x^2 = 4$

$3x^2 = 4$

$x^2 = 4/3$

$y = z = x = \pm\sqrt{4/3} = \pm\frac{2}{\sqrt{3}}$

1) $\frac{2}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}} + \frac{2}{\sqrt{3}} \cdot \frac{2}{\sqrt{3}} = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$

2) $-\frac{2}{\sqrt{3}} \cdot \left(-\frac{2}{\sqrt{3}}\right) + \left(-\frac{2}{\sqrt{3}} \cdot \left(-\frac{2}{\sqrt{3}}\right)\right) = \frac{4}{3} + \frac{4}{3} = \frac{8}{3}$