14.8

5. $f(x,y) = x^2 + y^2 \qquad 2x + 3y = 6$

$f_x = 2x \quad f_y = 2y \quad \langle 2x, 2y \rangle \quad g_x = 2 \quad g_y = 3 \quad \langle 2,3 \rangle$

$\langle 2x, 2y \rangle = \lambda \langle 2, 3 \rangle$

$2x = 2\lambda \qquad 2y = 3\lambda$

$4xy = 6(\lambda^2) \qquad y = \frac{3}{2}\lambda \qquad 4(\lambda)y = 6(\lambda^2)$

$x = \frac{6(\lambda)^2}{4y} \qquad y = \frac{3}{2}\lambda$

$x = \frac{6(\lambda)^2}{4(\frac{3}{2}\lambda)} = \frac{12(\lambda)^2}{12\lambda} = \lambda$

$2(\lambda) + 3(\frac{3}{2}\lambda) = 6$

$\frac{4}{2}\lambda + \frac{9}{2}\lambda = 6$

$\frac{13}{2}\lambda = 6$

$\lambda = \frac{12}{13} \qquad x = \frac{12}{13} \quad y = \frac{36}{26}$

$f(\frac{12}{13}, \frac{36}{26}) = (\frac{12}{13})^2 + (\frac{36}{26})^2 = \frac{144}{169} + \frac{\cancel{1296}\ 324}{\cancel{676}\ 169} = \frac{468}{169} = \frac{36}{13}$ min

7. $f(x,y) = xy \qquad 4x^2 + 9y^2 = 32$

$f_x = y \quad f_y = x \qquad g_x = 8x \quad g_y = 18y \qquad \langle y, x \rangle = \lambda \langle 8x, 18y \rangle$

$y = \lambda 8x \qquad x = \lambda 18y$

$xy = 144\lambda^2 xy$

$+\frac{1}{12}, -\frac{1}{12} = \lambda$

$4(\lambda 18y)^2 + 9(y)^2 = 32$

$1296\lambda^2 y^2 + 9y^2 = 32$

$9y^2(144\lambda^2 + 1) = 32$

$y = \sqrt{\frac{32}{9(144\lambda^2+1)}}$

$x = \sqrt{\frac{32}{18}} \qquad y = \sqrt{\frac{32}{8}}$

$4(x^2) + 9(\lambda 8x)^2 = 32$

$4x^2 + 576\lambda^2 x^2 = 32$

$4x^2(1 + 144\lambda^2) = 32$

$x = \sqrt{\frac{32}{4(1+144\lambda^2)}}$

max $= \frac{8}{3}$

min $= -\frac{8}{3}$

9. $f(x,y) = x^2 + y^2 \qquad x^4 + y^4 = 1$

$\langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle$

$2x = \lambda 4x^3 \qquad 2y = \lambda 4y^3$

$x^2 = \frac{1}{2\lambda} \qquad y^2 = \frac{1}{2\lambda}$

$\frac{1}{4\lambda^2} + \frac{1}{4\lambda^2} = 1$

$\frac{2}{4\lambda^2} = 1$

$2 = 4\lambda^2$

$\sqrt{\frac{1}{2}} = \lambda \qquad x = \sqrt{\frac{1}{2\sqrt{\frac{1}{2}}}} = \sqrt{\frac{\sqrt{2}}{2}} \qquad y = \sqrt{\frac{1}{2\sqrt{\frac{1}{2}}}} = \sqrt{\frac{1\sqrt{2}}{2}}$

$f\left(\sqrt{\frac{\sqrt{2}}{2}}, \sqrt{\frac{\sqrt{2}}{2}}\right) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2} \rightarrow$ maximum

$f\left(-\sqrt{\frac{\sqrt{2}}{2}}, -\sqrt{\frac{\sqrt{2}}{2}}\right) = 1 \rightarrow$ min

11. $f(x,y,z) = 3x + 2y + 4z \qquad x^2 + 2y^2 + 6z^2 = 1$

$\langle 3, 2, 4 \rangle = \lambda \langle 2x, 4y, 12z \rangle$

$3 = \lambda 2x \qquad 2 = \lambda 4y \qquad 4 = \lambda 12z$

$x = \frac{3}{2\lambda} \qquad y = \frac{1}{2\lambda} \qquad z = \frac{1}{3\lambda}$

$\left(\frac{3}{2\lambda}\right)^2 + 2\left(\frac{1}{2\lambda}\right)^2 + 6\left(\frac{1}{3\lambda}\right)^2 = 1$

$\frac{\cancel{9}\,81}{\cancel{4}\,36(\lambda)^2} + \frac{\cancel{2}\,18}{\cancel{4}\,36(\lambda)^2} + \frac{\cancel{6}\,24}{\cancel{9}\,36(\lambda)^2} = 1$

$\frac{123}{36(\lambda)^2} = 1$

$123 = 36(\lambda)^2$

$\sqrt{\frac{123}{36}} = \lambda$

$\sqrt{\frac{41}{12}} = \lambda \qquad x = \frac{3\sqrt{12}}{2\sqrt{41}} \qquad y = \frac{\sqrt{12}}{2\sqrt{41}} \qquad z = \frac{\sqrt{12}}{3\sqrt{41}}$

$f\left(\frac{3\sqrt{12}}{2\sqrt{41}}, \frac{\sqrt{12}}{2\sqrt{41}}, \frac{\sqrt{12}}{3\sqrt{41}}\right) = 3\left(\frac{3\sqrt{12}}{2\sqrt{41}}\right) + 2\left(\frac{\sqrt{12}}{2\sqrt{41}}\right) + 4\left(\frac{\sqrt{12}}{3\sqrt{41}}\right) = \frac{\cancel{9}^{18}\sqrt{12}}{\cancel{2}_{6}\sqrt{41}} + \frac{\cancel{2}^{6}\sqrt{12}}{\cancel{2}_{6}\sqrt{41}} + \frac{\cancel{4}^{8}\sqrt{12}}{\cancel{3}_{6}\sqrt{41}} = \frac{32\sqrt{12}}{6\sqrt{41}}$

max = 3.7 $\qquad$ min = −3.7

13. $f(x,y,z) = xy + 2z \qquad x^2 + y^2 + z = 0$

$\langle y, x, 2 \rangle = \lambda \langle 2x, 2y, 1 \rangle$

$y = \lambda 2x \qquad x = \lambda 2y \qquad 2 = \lambda 1$

$2 = \lambda$

$(x)^2 + (\lambda 2x)^2 = 0 \qquad (\lambda 2y)^2 + (y^2) = 0$

$x^2(1 + \lambda 2) = 0 \qquad y^2(1 + \lambda 2) = 0$

$x = 0 \qquad y = 0$

15. $f(x,y,z) = xy + xz \qquad x^2 + y^2 + z^2 = 4$

$x(y+z) = x(0) + 1(y+z)$

$\langle y+z, x, x \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$y + z = \lambda 2x \qquad x = \lambda 2y \qquad x = \lambda 2z \qquad \lambda 2y = \lambda 2z$

$y = z$

$x^2(y+z) = (\lambda)^3 8xyz$

$(\lambda 2y)^2 (2y) = (\lambda)^3 8xy^2$

$4\cancel{(\lambda)^2} 2y^{\cancel{3}} = (\lambda)^{\cancel{3}} 8x\cancel{y^2}$

$8y = \lambda 8(\lambda 2y)$

$\frac{1}{2\lambda^2} = \lambda^2$

$1 = 2\lambda^4$

$\frac{1}{2} = \lambda^4$

$\sqrt[4]{\frac{1}{2}} = \lambda$

$(\lambda 2y)^2 + \left(\frac{x}{\lambda 2}\right)^2 + \left(\frac{x}{\lambda 2}\right)^2 = 4$

$4\lambda^2 y^2 + \frac{2x^2}{4\lambda^2} = 4$

$xy + xz = \left(\sqrt[4]{\frac{1}{2}} \cdot 2 \cdot \frac{x}{\lambda 2}\right)\left(\frac{x}{\lambda 2}\right) + \left(\lambda 2 \sqrt[4]{\frac{1}{2}}\right)\left(\frac{x}{\lambda 2}\right) = 2\sqrt{2}$ max

$= -2\sqrt{2}$ min