Fayed Raza
10/18/2020

14.8: 5, 7, 9, 11, 13, 15

5. $f(x,y) = x^2 + y^2$, $2x+3y=6$

$f_x = \lambda g_x$ $\quad f_y = \lambda g_y$

$2x = \lambda 2$ $\quad 2y = \lambda 3$

$x = \lambda$ $\quad y = \lambda \frac{3}{2}$

$2x + \frac{3}{2}(x)$

$2x + 3\left(\frac{3}{2}x\right) = 6$

$2x + \frac{9}{2}x = 6$

$\frac{13}{2}x = 6$

$x = \frac{12}{13}$

$y = \frac{36}{26} = \frac{18}{13}$

$\frac{144}{169} + \frac{324}{169}$

$\frac{468}{169}$

Minimum $= \frac{36}{13}$

$x = \frac{12}{7}$

$y = \frac{36}{14} = \frac{18}{7}$

$2\left(\frac{12}{7}\right) + 3y = 6$

$\frac{24}{7} + 3y = 6$

$3y = \frac{18}{7}$

$y = \frac{6}{7}$

$\frac{144}{49} + \frac{36}{49} = \frac{180}{49}$

7. $f(x) = xg(x)$

$y = \pm 8x \qquad x = \pm 18y$

$yx = x^2 (18)(8) \; xy$

$y = \frac{x^2}{\sqrt{(18)(8)}} \qquad y = \frac{8}{12}x$

$y = \frac{x^2}{12} \qquad x = \frac{12}{8}y$

$x = \frac{3}{2}y$

$4(\frac{3}{2}y)^2 + 9y^2 = 32$

$4(\frac{9}{4})x^2 + 9y^2 = 32$

$9y^2 + 9y^2 = 32$

$18y^2 = 32$

$y^2 = \frac{32}{18}$

$y = \frac{\sqrt{16}\sqrt{2}}{\sqrt{9}\sqrt{2}}$

$y = \frac{4\sqrt{2}}{3\sqrt{2}}$

$y = \pm\frac{4}{3}$

$4x^2 + 9(\frac{64}{144})x^2 = 32$

$4x^2 + 4x^2 = 32$

$8x^2 = 32$

$x = \pm 2$

$x = \pm\frac{1}{12}$

$\max = 2(\frac{4}{3}) - \frac{8}{3}$

$\min = 2(-\frac{4}{3}) = -\frac{8}{3}$

9. $f(x,y) = x^2 + y^2$, $x^4 + y^4 = 1$

$\langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle$

$2x = \lambda 4x^3$ $\quad$ $2y = \lambda 4y^3$

$\frac{2}{4x^2} = \lambda^2$ $\quad$ $x = \lambda = 2y^3$

$\sqrt{\frac{1}{2x}} = x$ $\quad$ $\frac{1}{y^2} = 2x^2$

$y = \sqrt{\frac{1}{2x}}$

$\frac{1}{4x^2} + \frac{1}{4x^2} = 1$ $\quad$ $\frac{4}{4x^2} = \frac{y^2}{2}$ $\quad$ $\frac{1}{x^2} \cdot 2$

$f\left(\sqrt{\frac{1}{2\sqrt{2}}}, \sqrt{\frac{1}{2\sqrt{2}}}\right) \frac{2}{4x^2} =$ $\quad$ $\frac{2}{4} = \frac{y^2}{x^2}$ $\quad$ $x = \pm\sqrt{\frac{1}{2}}$

$2\left(\sqrt{\frac{1}{2\sqrt{2}}}\right)^2 =$ $\quad$ $\pm\sqrt{\frac{1}{2}} = \sqrt{x^2}$

$2\,\frac{1}{2\sqrt{2}}$ $\quad$ $\frac{1}{\sqrt{2}}$ $\quad$ $\sqrt{2}$

Max: $\sqrt{2}$
Min: -1

$f\left(\sqrt{\frac{1}{2-\sqrt{2}}}, \sqrt{\frac{1}{2+\sqrt{2}}}\right) = -1$

11. $f(x,y,z) = 3x+2y+4z,\ x^2+2y^2+6z^2=1$

$\langle 3,2,4\rangle = \lambda\langle 2x, 4y, 12z\rangle$

$3 = \lambda 2x \quad 3 = \lambda 4y \quad y = \lambda 12z$

$\frac{3}{\lambda 2} = x \quad \frac{3}{4\lambda} = \lambda \quad \frac{4}{\lambda 12} = z$

$x = \frac{3}{4} \quad y = \frac{3}{8} \quad z = \frac{4}{24} = \frac{1}{6}$

$\frac{9}{4\lambda^2} + \frac{9}{16\lambda^2} + \frac{6}{\lambda^2 9} = 1$

$\frac{2,25}{\lambda^2} + \frac{1,125}{\lambda^2} + \frac{0,67}{\lambda^2} = 1$

$\lambda = \pm 3,7$

Max: 3,7

min: -3,7

13 $f(x,y,z) = xyz$ subject to $x^2+y^2+z^2 = 36$

$f(4,4,\frac{1}{2}) = 16+2 = 20$

$f(-4,-4,\frac{1}{2}) = 16+2 = 20$

$\langle yz, xz, xy \rangle = \lambda \langle 2x, 2y, 2z \rangle$

Max: 20
Min: -20

$yz = \lambda 2x$ $\quad xz = \lambda 2y$ $\quad xy = \lambda 2z$

$\frac{xyz}{xyz} = \lambda^3 \frac{8xyz}{xyz}$ $\quad 1 = \lambda^3 = 8$ $\quad \lambda = \frac{1}{2}$

$\frac{x}{y} = \frac{y}{x}$ $\quad x^2 = y^2$ $\quad x = y$

$z = \frac{1}{x}$

$x^2 + x^2 + z^2 = 36$ $\quad 2x^2 + \frac{1}{x^2} = 36$ $\quad 2x^4 + 1 = 36x^2$

$2x^4 - 36x^2 + 1 = 0$ $\quad x^2 = 16$ $\quad x = \pm 4$

$z = \frac{1}{4}$

15 $f_x = \lambda g_x$ $\quad f_y = \lambda g_y$

$\langle y+z, x, x \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$(y+z) = \lambda(2x)$, $\quad x = \lambda(2y)$,

$f_z = \lambda g_z$

$x = \lambda(2z)$

$\frac{x}{2x} = \lambda$

$f(\sqrt{\frac{1}{2}}, \frac{1}{2}, \frac{1}{2}) = 2$

$f(-\sqrt{\frac{1}{2}}, \frac{1}{2}, \frac{1}{2}) =$

$x = \pm\sqrt{\frac{1}{2}}$

$\frac{x}{2x} = z$

$\frac{x^2}{4x^2} + \frac{x^2}{4x^2} + x^2 = 4$ $\quad z = \frac{1}{2}$

$\frac{2x^2}{4x^2} + x^2 = y$ $\quad y = \frac{1}{2}$

$\frac{x^2}{2x^2} + x^2 = 4$ $\quad x^2 = 4 - \frac{1}{2}$ $\quad x = \pm\sqrt{\frac{7}{2}}$

Max: $2\sqrt{2}$
Min: $-2\sqrt{2}$