14.8 Homework

⑤ $f(x,y) = x^2+y^2$, $2x+3y=6$

→ $\nabla f = \langle 2x, 2y \rangle$, $\nabla g = \langle 2, 3 \rangle$

→ $\langle 2x, 2y \rangle = \langle 2\lambda, 3\lambda \rangle$

→ $2x = 2\lambda$, $2y = 3\lambda$ ⇒ $x = \lambda$, $y = \frac{3}{2}\lambda$

→ $2\lambda + 3 \cdot \frac{3}{2}\lambda = 6$ ⇒ $2\lambda + \frac{9}{2}\lambda = 6$ ⇒ $4\lambda + 9\lambda = 12$ ⇒ $13\lambda = 12$ ⇒ $\lambda = \frac{12}{13}$

→ $x = \frac{12}{13}$, $y = \frac{18}{13}$

→ $f\left(\frac{12}{13}, \frac{18}{13}\right) = \frac{144}{169} + \frac{324}{169} =$ $\frac{36}{13}$ is the maximum value

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⑦ $f(x,y) = xy$, $4x^2+9y^2 = 32$

→ $\nabla f = \langle y, x \rangle$, $\nabla g = \langle 8x, 18y \rangle$

→ $\langle y, x \rangle = \langle 8\lambda x, 18\lambda y \rangle$

→ $y = 8\lambda x$, $x = 18\lambda y$

→ Critical Point: $(\pm 2, \pm \frac{4}{3})$

→ Max is $\frac{8}{3}$ and min. is $-\frac{8}{3}$

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⑨ $f(x,y) = x^2+y^2$, $x^4+y^4=1$

→ $\nabla f = \langle 2x, 2y \rangle$, $\nabla g = \langle 4x^3, 4y^3 \rangle$

→ $\langle 2x, 2y \rangle = \langle 4\lambda x^3, 4\lambda y^3 \rangle$

→ $2x = 4\lambda x^3$, $2y = 4\lambda y^3$

→ $\lambda = \frac{1}{2x^2}$, $\lambda = \frac{1}{2y^2}$

→ $2x^2 = 2y^2$ ⇒ $x^2 = y^2$ ⇒ Critical Points: $\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right)$, $(0, \pm 1)$, $(\pm 1, 0)$

→ $f\left(\pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) = \frac{2}{\sqrt{2}}$

→ $f(0,-1) = 1$, $f(0,1) = 1$, $f(-1,0) = 1$, $f(1,0) = 1$

→ Max is $\frac{2}{\sqrt{2}}$ and min is 1

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⑪ $f(x,y,z) = 3x+2y+4z$, $x^2+2y^2+6z^2 = 1$

→ $\nabla f = \langle 3, 2, 4 \rangle$, $\nabla g = \langle 2x, 4y, 12z \rangle$

→ $\langle 3, 2, 4 \rangle = \langle 2\lambda x, 4\lambda y, 12\lambda z \rangle$

→ $\lambda = \frac{3}{2x}$, $\lambda = \frac{1}{2y}$, $\lambda = \frac{1}{3z}$

→ Critical Points: $\left(\pm 3\sqrt{\frac{3}{41}}, \pm \sqrt{\frac{3}{41}}, \pm \frac{2}{\sqrt{123}} \right)$

→ Max is 3.7 and min is -3.7

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(13) $f(x,y) = xy + 2z$, $x^2 + y^2 + z^2 = 36$

→ $\nabla f = \langle y, x, 2 \rangle$, $\nabla g = \langle 2x, 2y, 2z \rangle$

→ $\langle y, x, 2 \rangle = \langle 2\lambda x, 2\lambda y, 2\lambda z \rangle$

→ Critical Points: $(\pm 4, \pm 4, 2)$

→ Max is 20 and min is -20
