Sec. 14.8

5) $\nabla f = \langle 2x, 2y \rangle$ $\nabla g = \langle 2, 3 \rangle$

$\langle 2x, 2y \rangle = \lambda \cdot \langle 2, 3 \rangle$

$2x = 2\lambda$ $2y = 3\lambda$ $2x + 3y = 6$

$x = \lambda$ $y = \frac{3}{2}\lambda$ $2\lambda + \frac{9}{2}\lambda = 6$

$x = \frac{12}{13}$ $y = \frac{18}{13}$ $\frac{13}{2}\lambda = 6$

$f\left(\frac{12}{13}, \frac{18}{13}\right) = \frac{36}{13}$ $\lambda = \frac{12}{13}$

Min: $\frac{36}{13}$ Max: none

7) $\nabla f = \langle y, x \rangle$ $\nabla g = \langle 8x, 18y \rangle$

$\langle y, x \rangle \lambda = \langle 8x, 18y \rangle$

$y\lambda = 8x \qquad x\lambda = 18y \qquad 4x^2 + 9y^2 = 32$

$(x,y) = (2, 4/3), \ (-2, 4/3), \ (2, -4/3), \ (-2, -4/3)$

$f(2, 4/3) = \dfrac{8}{3} \qquad f(-2, 4/3) = -\dfrac{8}{3} \qquad f(2, -4/3) = -\dfrac{8}{3}$

$f(-2, -4/3) = \dfrac{8}{3} \qquad Min = -\dfrac{8}{3} \qquad max = \dfrac{8}{3}$

9) $\nabla f = \langle 2x, 2y \rangle \qquad \nabla g = \langle 4x^3, 4y^3 \rangle$

$\langle 2x, 2y \rangle \cdot \lambda = \langle 4x^3, 4y^3 \rangle$

$2x\lambda = 4x^3 \quad 2y\lambda = 4y^3 \qquad x^4 + y^4 = 1$

using maple; $f(0,1) = 1 \qquad f(2^{-1/4}, 2^{-1/4}) = \sqrt{2}$

$Min = 1 \qquad\qquad max = \sqrt{2}$

11) $\nabla f = \langle 3, 2, 4 \rangle \qquad \nabla g = \langle 2x, 4y, 12z \rangle$

$\langle 3, 2, 4 \rangle \cdot \lambda = \langle 2x, 4y, 12z \rangle$

$3\lambda = 2x \quad 2\lambda = 4y \quad 4\lambda = 12z \quad x^2 + 2y^2 + 6z^2 = 1$

$f\left(\dfrac{3}{41}\sqrt{123}, \ \dfrac{1}{41}\sqrt{123}, \ \dfrac{2}{123}\sqrt{123}\right) = \dfrac{\sqrt{123}}{3}$

$$f\left(-\frac{3}{41}\sqrt{123}, -\frac{1}{41}\sqrt{123}, -\frac{2}{123}\sqrt{123}\right) = -\frac{\sqrt{123}}{3}$$

13) $\nabla f = \langle y, x, 2 \rangle$ $\nabla g = \langle 2x, 2y, 2z \rangle$

$$\langle y, x, 2 \rangle \cdot \lambda = \langle 2x, 2y, 2z \rangle$$

$\lambda y = 2x$ $\lambda x = 2y$ $2\lambda = 2z$ $x^2 + y^2 + z^2 = 36$

$\quad f(4, 4, 2) = 20$ $f(4, -4, -2) = -20$

max $= 20$ min $= -20$

15) $\nabla f = \langle y+z, x, x \rangle$ $\nabla g = \langle 2x, 2y, 2z \rangle$

$$\langle y+z, x, x \rangle \cdot \lambda = \langle 2x, 2y, 2z \rangle$$

$\lambda(y+z) = 2x$ $\lambda x = 2y$ $\lambda x = 2z$ $x^2 + y^2 + z^2 = 4$

$\quad f(\sqrt{2}, 1, 1) = 2\sqrt{2}$ $f(-\sqrt{2}, 1, 1) = -2\sqrt{2}$

max $= 2\sqrt{2}$ min $= -2\sqrt{2}$