14.8 HW - # 5, 7, 9, 11, 13, 15

5 $f(x,y) = x^2 + y^2$

$g(x,y) = 2x + 3y = 6$

$\nabla f = \langle 2x, 2y \rangle$

$\nabla g = \langle 2, 3 \rangle$

$\nabla f = \lambda \nabla g$

$\langle 2x, 2y \rangle = \langle 2\lambda, 3\lambda \rangle$

$2x = 2\lambda \rightarrow \lambda = x$

$2y = 3x \rightarrow \lambda = \frac{2}{3}y$

$\rightarrow x = \frac{2}{3}y$

$3x = 2y$

$y = \frac{3x}{2}$

$3y + 2x = 6$

$3y + 2\left(\frac{2}{3}y\right) = 6$

$3y + \frac{4}{3}y = 6$

$\frac{13}{3}y = 6$

$13y = 18$

$y = \frac{13}{18}$

$\rightarrow x = \frac{2}{3}\left(\frac{13}{18}\right)$

$x = \frac{26}{54}$

$x = \frac{13}{27}$

$(x,y) = \left(\frac{13}{27}, \frac{13}{18}\right)$

9. $f(x,y) = x^2 + y^2$

$g(x,y) = x^4 + y^4 = 1$

$\nabla f = (2x, 2y)$

$\nabla g = (4x^3, 4y^3)$

$\langle 2x, 2y \rangle = \langle 4x^3\lambda, 4y^3\lambda \rangle$

$2x = 4x^3\lambda$

$\hookrightarrow \frac{1}{2x^2} = \lambda$

$2y = 4y^3\lambda$

$\hookrightarrow \frac{1}{2y^2} = \lambda$

$\rightarrow 2x^2 = 2y^2$

$x^2 = y^2$

$x^4 + x^4 = 1$

$2x^4 = 1 \Rightarrow x^4 = \frac{1}{2}$

$x = \pm\sqrt[4]{\frac{1}{2}}$

min $= \left(-\sqrt[4]{\frac{1}{2}}, -\sqrt[4]{\frac{1}{2}}\right)$

max $= \left(\sqrt[4]{\frac{1}{2}}, \sqrt[4]{\frac{1}{2}}\right)$

7. $f(x,y) = xy$

$g(x,y) = 4x^2 + 9y^2 = 32$

minimum: $\left(\sqrt{\frac{36}{5}}, \sqrt{\frac{16}{45}}\right)$

maximum: $\left(-\sqrt{\frac{36}{5}}, -\sqrt{\frac{16}{45}}\right)$

$\nabla f = (y, x)$

$\nabla g = \langle 8x, 18y \rangle$

$\nabla f = \lambda \nabla g$

$\langle y, x \rangle = \langle 8x, 18y \rangle \lambda$

$y = 8x\lambda \rightarrow \lambda = \frac{y}{8x}$

$x = 18y\lambda \rightarrow \lambda = \frac{x}{18y}$

$\rightarrow \frac{y}{8x} = \frac{x}{18y}$

$18y^2 = 8x^2 \rightarrow y^2 = \frac{8}{18}x^2$

$4x^2 + 9y^2 = 32$

$4x^2 + \frac{8}{18}x^2 = 32$

$\frac{170}{18}x^2 = 32$

$y = \pm\sqrt{\frac{16}{45}}$

$4\left(\frac{36}{5}\right) + 9y^2 = 32$

$\frac{144}{5}$ $\quad 9y^2 = \frac{16}{5}$

$\begin{cases} x = \pm\sqrt{\frac{36}{5}} \\ x = \pm\sqrt{\frac{576}{80}} \end{cases}$

$f(x,y,z) = 3x+2y+4z$

$g(x,y,z) = x^2+2y^2+6z^2=1$

$\nabla f = (3,2,4)$

$\nabla g = \langle 2x, 4y, 12z\rangle$

$\nabla f = \lambda \nabla g$

$\langle 3,2,4\rangle = \langle 2x\lambda, 4y\lambda, 12z\lambda\rangle$

$3 = 2x\lambda \rightarrow \lambda = \frac{3}{2}x$

$2 = 4y\lambda \rightarrow \lambda = \frac{1}{2}y$

$\frac{3}{2}x = \frac{1}{2}y \rightarrow x = \frac{1}{3}y$

$4 = 12z\lambda \rightarrow \lambda = \frac{1}{3}z$

$\frac{1}{2}y = \frac{1}{3}z \rightarrow z = \frac{3}{2}y$

$(\frac{1}{3}y)^2 + 2y^2 + 6(\frac{3}{2}y)^2 = 1$

$\frac{1}{9}y^2 + 2y^2 + 9y^2 = 1$

$\frac{100}{9}y^2 = 1$

$y = \pm\sqrt{\frac{9}{100}}$

min: $\left(-\frac{1}{3}\sqrt{\frac{9}{100}}, -\sqrt{\frac{9}{100}}, -\frac{3}{2}\sqrt{\frac{9}{100}}\right)$

max: $\left(\frac{1}{3}\sqrt{\frac{9}{100}}, \sqrt{\frac{9}{100}}, \frac{3}{2}\sqrt{\frac{9}{100}}\right)$

13. $f(x,y,z) = xy+2z$

$g(x,y,z) = x^2+y^2+z^2 = 36$

$\nabla f = \langle y, x, 2\rangle$

$\nabla g = \langle 2x, 2y, 2z\rangle$

max = 20

min = -20

$y = 2x\lambda \rightarrow y = 2xz \rightarrow \frac{y}{2z} = x$

$x = 2y\lambda \rightarrow x = 2yz$

$2yz = \frac{y}{2z}$

$2 = 2z\lambda$

$4z^2y = y$

$z = \lambda$

$4z^2 = 1$

$x^2+y^2+(-\frac{1}{2})^2 = 36$

$z = \pm\sqrt{\frac{1}{4}}$

$x^2+y^2+\frac{1}{4} = 36$

$z = \pm\frac{1}{2}$

15. $f(x,y,z) = xy + xz$

$g(x,y,z) = x^2 + y^2 + z^2 = 4$

$\nabla f = \langle y+z, x, x \rangle$

$\nabla g = \langle 2x, 2y, 2z \rangle$

max: $2\sqrt{2}$

min: $-2\sqrt{2}$

$\nabla f = \lambda \nabla g$

$y + z = 2x\lambda \rightarrow 2z = 2x\lambda \rightarrow x = \frac{z}{\lambda} \rightarrow x = z\left(\frac{2y}{x}\right)$

$x = 2y\lambda$
$x = 2z\lambda$
$\Big] \; 2y\lambda = 2z\lambda$

$y = z$

$x = \frac{2yz}{\lambda}$

$\lambda = \frac{x}{2y}$

$\frac{z^2}{\lambda^2} + z^2 + z^2 = 4$

$\frac{z^2}{\lambda^2} + 2z^2 = 4$

$\frac{2z^2}{\lambda} = \frac{z^2}{\lambda^2}$

$2z^2\lambda^2 = z^2\lambda$

$2\lambda = 1$

$\lambda = \frac{1}{2}$

$z^2 + 2z^2 = 4$

$3z^2 = 4$

$z = \pm\sqrt{\frac{4}{3}}$