14.8 Homework

5. $f(x,y) = x^2 + y^2$ $\quad g(x,y) = 2x + 3y = 6$

$\nabla f = \langle 2x, 2y \rangle$ $\quad \nabla g = \langle 2, 3 \rangle$

$\nabla f = \lambda \nabla g$

$\langle 2x, 2y \rangle = \lambda \langle 2, 3 \rangle$

$\rightarrow 2x = \lambda 2, \quad 2y = \lambda 3$

$2x + 3y = 6$

$x = \lambda \quad y = \frac{\lambda 3}{2}$

$2(\lambda) + 3\left(\frac{\lambda 3}{2}\right) = 6$

$2\lambda + \frac{9\lambda}{2} = 6$

$\frac{13\lambda}{2} = 6 \qquad \lambda = 12/13$

$x = 12/13 \quad y = 18/13$

$f(12/13, 18/13) = 36/13$

$\therefore$ The minimum value is $36/13$.

7. $f(x,y) = xy$ $\quad g(x,y) = 4x^2 + 9y^2 - 32$

$\nabla f = \langle y, x \rangle$ $\quad \nabla g = \langle 8x, 18y \rangle$

$\nabla f = \lambda \nabla g$

...

$y = 8\lambda x, \quad x = 18\lambda y$

$4x^2 + 9y^2 - 32$

$\lambda = \frac{y}{8x} \qquad \lambda = \frac{x}{18y}$

$\frac{y}{8x} = \frac{x}{18y}$

$18y^2 = 8x^2$

$y = \pm 2/3\, x$

$4x^2 + 9(\pm 2/3\, x)^2 = 32$

$4x^2 \pm 4x^2 = 32$

$x = \pm 2$

Critical Points: $(2, 4/3), (-2, -4/3)$

$f(2, 4/3) = 8/3$

$f(-2, -4/3) = -8/3$

Maximum val: $8/3$, Minimum Value: $-8/3$

9. $f(x,y) = x^2 + y^2 \qquad g(x,y) = x^4 + y^4 - 1$

$\nabla f = \langle 2x, 2y \rangle \qquad \nabla g = \langle 4x^3, 4y^3 \rangle$

$\langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle$

$2x = 4\lambda x^3 \qquad 2y = 4\lambda y^3, \quad x^4 + y^4 - 1$

$\lambda = \frac{1}{2x^2} \qquad \lambda = \frac{1}{2y^2}$

$\frac{1}{2x^2} = \frac{1}{2y^2} \Rightarrow x^2 = y^2$

$x^4 + x^4 - 1 = 0$

$2x^4 = 1 \rightarrow x = \pm \frac{1}{4\sqrt{2}} \quad y = \pm \frac{1}{4\sqrt{2}}$

$f(1/4\sqrt{2}, 1/4\sqrt{2}) = \sqrt{2}$

$f(-1/4\sqrt{2}, -1/4\sqrt{2}) = \sqrt{2}$

Min/Max $= \sqrt{2}$

11. $\nabla f = \langle 3, 2, 4 \rangle \quad \nabla g = \langle 2x, 4y, 12z \rangle$

$\langle 3, 2, 4 \rangle = \lambda \langle 2x, 4y, 12z \rangle$

$3 = \lambda 2x \quad 2 = \lambda 4y \quad 4 = \lambda 12z, \quad x^2 + 2y^2 + 6z^2 - 1$

$\lambda = \frac{3}{2x} \quad \lambda = \frac{1}{2y} \quad \lambda = \frac{1}{3z}$

$\frac{3}{2x} = \frac{1}{2y} = \frac{1}{3z}$

$y = \frac{x}{3} \quad z = \frac{2x}{9}$

$x^2 + \left(\frac{x}{3}\right)^2 + \left(\frac{2x}{9}\right)^2 - 1 = 0$

$x^2 + \frac{x^2}{9} + \frac{4x^2}{81} - 1 = 0$

$x = \pm 9/\sqrt{123}$

Critical Points: $(9/\sqrt{123}, 3/\sqrt{123}, 2/\sqrt{123}) = 41/\sqrt{123}$ = max

$(-9/\sqrt{123}, -3/\sqrt{123}, -2/\sqrt{123}) = -41/\sqrt{123}$ = min

13. $\nabla f = \langle y, x, 2 \rangle \qquad \nabla g = \langle 2x, 2y, 2z \rangle$

$y = \lambda 2x \qquad x = \lambda 2y \qquad 2 = \lambda 2z$

$x^2 + y^2 + z^2 - 36$

$x^2 + (\lambda 2x)^2 + (\lambda 2z)^2$?

Not good with solving these.

$(\lambda 2x)(\lambda 2y)(\lambda 2z) = xyz$

$\lambda^3 8xyz = xyz$

$\lambda^3 8 = 0$

15. $f(x,y,z) = xy + xz, \quad x^2 + y^2 + z^2 = 4$

$\nabla f = \langle y+z, x, x \rangle \qquad \nabla g = \langle 2x, 2y, 2z \rangle$

$y + z = \lambda 2x$

$x = \lambda 2y$

$x = \lambda 2z$

$(\lambda 2y)^2 + y^2 + (\lambda 2x - y)^2 = 4$

$\lambda^2 4y^2 + y^2 + (\lambda 2(\lambda 2y) - y)^2 = 4$

$\lambda^2 4y^2 + y^2 + (\lambda 2 \lambda 2y - y)^2 = 4$

$\lambda^2 4y^2 + y^2 + (4\lambda^2 - y)^2 = 4$

$$(y+z)(x)(x) = (\lambda 2x)(\lambda 2y)(\lambda 2z)$$

$$(y+z)(x)^2 = \lambda^3 8xyz$$

$$yx^2 + zx^2 = \lambda^3 8xyz$$