14.8 HW

$\nabla f = \lambda \nabla g$

5. $f(x,y) = x^2+y^2$, $2x+3y=6$ $\nabla f = \langle 2x, 2y\rangle$ $\nabla g = \langle 2, 3\rangle$

$2x = 2\lambda$ $2y = 3\lambda$ $2(\lambda) + 3(\frac{3}{2}\lambda) = 6$

$x = \lambda$ $y = \frac{3}{2}\lambda$ $2\lambda + \frac{9}{2}\lambda = 6$

$x = 12/13$ $y = 18/13$ $\frac{13}{2}\lambda = 6$ $\lambda = 12/13$

$f(x,y) = (12/13)^2 + (18/13)^2 = \boxed{36/13}$

7. $f(x,y) = xy$, $4x^2+9y^2=32$ $\nabla f = \langle y, x\rangle$ $\nabla g = \langle 8x, 18y\rangle$

$y = 8\lambda x$ $x = 18\lambda y$ $\frac{y}{8x} = \frac{x}{18y}$ $2x = \pm 3y$

$(\pm 2, \pm 4/3)$ $4x^2+9y^2-32=0$

$f(-2, 4/3) = -8/3$ $f(2, 4/3) = 8/3$ $x = \pm 4$ $y = \pm 2/3 x$

$\boxed{\text{Max} = 8/3 \quad \text{Min} = -8/3}$

9. $f(x,y) = x^2+y^2$, $x^4+y^4=1$ $\nabla f = \langle 2x, 2y\rangle$ $\nabla g = \langle 4x^3, 4y^3\rangle$

$2x = 4\lambda x^3$ $2y = 4\lambda y^3$ $x = \pm \frac{1}{(2)^{1/4}}$ $y = \pm \frac{1}{(2)^{1/4}}$

$(\pm \frac{1}{2^{1/4}}, \pm \frac{1}{2^{1/4}})$ $(\pm 1, 0)$ $(0, \pm 1)$ $\boxed{\text{Max} = \sqrt{2} \quad \text{Min} = 1}$

11. $f(x,y,z) = 3x+2y+4z$, $x^2+2y^2+6z^2=1$ $\nabla f = \langle 3, 2, 4\rangle$ $\nabla g = \langle 2x, 4y, 12z$

$3 = 2\lambda x$ $2 = 4\lambda y$ $4 = 12\lambda z$

$x = \frac{3}{2\lambda}$ $y = \frac{1}{2\lambda}$ $z = \frac{1}{3\lambda}$ $x^2+2y^2+6z^2=1$

$\frac{9}{4\lambda^2} + \frac{2}{4\lambda^2} + \frac{6}{9\lambda^2} = 1$

$\frac{324}{4\lambda^2} + \frac{72}{4\lambda^2} + \frac{216}{9\lambda^2} = 36$ $\frac{81}{\lambda^2} + \frac{18}{\lambda^2} + \frac{24}{\lambda^2} = 36$ $123 = 36\lambda^2$

$\lambda = 1.85$: $x = 0.81$ $y = 0.27$ $z = 0.18$ $\lambda^2 = 3.41$ $\lambda = \pm 1.85$

$\lambda = -1.85$: $x = -0.81$ $y = -0.27$ $z = -0.18$

$\boxed{\text{Max: } 3.7 \quad \text{Min: } -3.7}$

13. $f(x,y,z) = xy + 2z$, $x^2+y^2+z^2 = 36$ $\nabla f = \langle y, x, 2\rangle$ $\nabla g = \langle 2x, 2y, 2z\rangle$

$y = 2\lambda x$ $x = 2\lambda y$ $2 = 2\lambda z$ $y(1-4\lambda^2) = 0$

$x^2+y^2+z^2-36=0$ $x^2+(\pm x)^2+(\pm 2)^2-36=0$ $x = \pm y$

$2x^2 = 32$

$x^2 = 16$

$x = \pm 4$ critical points: $(\pm 4, \pm 4, \pm 2)$ and $(0,0,\pm 6)$

$f(4,4,2) = f(-4,-4,2) = 20$ $f(4,-4,2) = f(-4,4,-2) = -20$

$f(4,4,-2) = f(-4,-4,-2) = 12$ $f(4,-4,2) = f(-4,4,2) = -12$

$f(0,0,\pm 6) = \pm 12$ Max is 20, min is -20

15. $f(x,y,z) = xy + xz$, $x^2+y^2+z^2 = 4$ $\nabla f = \langle y+z, x, x\rangle$ $\nabla g = \langle 2x, 2y, 2z\rangle$

$y+z = 2\lambda x$ $x = 2\lambda y$ $x = 2\lambda z$ $x^2 = 2y^2$

$x+y^2+z^2-4=0 \rightarrow 2y^2+y^2+y^2-4=0$ $4y^2 = 4$ $y = \pm 1$

points: $(\pm\sqrt{2}, 1, 1)$ and $(\pm\sqrt{2}, -1, -1)$

critical values: $2\sqrt{2}$, $-2\sqrt{2}$, 0 Max $= 2\sqrt{2}$ min $= -2\sqrt{2}$