Afsana Rahman

Dr Z - Calc 3

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HW 6 - 14.8

$\boxed{14.8}$ 5) $f(x,y) = x^2 + y^2$

$\nabla f = \langle 2x, 2y \rangle$

$g(x,y) = 2x + 3y$

$\nabla g = \langle 2, 3 \rangle$

$2x = 2\lambda,\ 2y = 3\lambda$

$x = \lambda,\ y = \frac{3}{2}\lambda$

$2\lambda + 3(\frac{3}{2}\lambda) = 6$

$13\lambda = 12,\ \lambda = \frac{12}{13}$

$x = \frac{12}{13},\ y = \frac{3}{2}(\frac{12}{13})$

$x^2 + y^2 = (\frac{12}{13})^2 + (\frac{18}{13})^2$

$= \frac{468}{169} = \frac{36}{13}$

7) $f(x,y) = xy,\ \nabla f = \langle y, x \rangle$

$g(x,y) = 4x^2 + 9y^2,$

$\nabla g = \langle 8x, 18y \rangle$

$\lambda y = 8x,\ \lambda x = 18y$

$y = \frac{8x}{\lambda},\ x = \frac{18(8x)}{\lambda^2}$

$\lambda^2 = 18 * 8,\ \lambda = \pm 12$

$4x^2 + 9(\frac{2x}{3})^2 = 32$

$4x^2 + 4x^2 = 32,\ x = \pm 2$

$y = \frac{8(2)}{12} = \frac{4}{3}$

$max = 2(\frac{4}{3}) = \frac{8}{3},\ min = -\frac{8}{3}$

9) $f(x,y) = x^2 + y^2,\ \nabla f = \langle 2x, 2y \rangle$

$g(x,y) = x^4 + y^4,\ \nabla g = \langle 4x^3, 4y^3 \rangle$

$\lambda 2x = 4x^3,\ \lambda 2y = 4y^3$

$\lambda = 2x^2 = 2y^2,\ x = y = \sqrt{\frac{\lambda}{2}}$

$x^4 + y^4 = 2(\sqrt{\frac{\lambda}{2}})^4 = 2(\frac{\lambda^2}{4}) = 1$

$\lambda^2 = 2,\ \lambda = \pm\sqrt{2},\ x = y = \pm\frac{\sqrt{\sqrt{2}}}{\sqrt{2}}$

$max = \sqrt{2},\ min = 0$

11) $f(x,y,z) = 3x + 2y + 4z,\ \nabla f = \langle 3, 2, 4 \rangle$

$g(x,y,z) = x^2 + 2y^2 + 6z^2,\ \nabla g = \langle 2x, 4y, 12z \rangle$

$3\lambda = 2x,\ 2\lambda = 4y\ \ 4\lambda = 12z$

$x = \frac{3}{2}\lambda,\ y = \frac{1}{2}\lambda,\ z = \frac{1}{3}\lambda$

$(\frac{3}{2}\lambda)^2 + 2(\frac{1}{2}\lambda)^2 + 6(\frac{1}{3}\lambda)^2 = 1$

$\frac{9}{4}\lambda^2 + \frac{2}{4}\lambda^2 + \frac{6}{9}\lambda^2 = 1$

$\lambda^2(81 + 18 + 24) = 36$

$123\lambda^2 = 36\ \ \lambda = \pm\frac{\sqrt{123}}{6}$

$x = \frac{1}{4}\sqrt{123},\ y = \frac{1}{12}\sqrt{123},\ z = \frac{1}{18}\sqrt{123}$

$\sqrt{123}(\frac{3}{4} + \frac{2}{12} + \frac{4}{18}) = \pm\frac{41\sqrt{123}}{36}$

13) $f(x,y,z) = xy + 2z,\ \nabla f = \langle y, x, 2 \rangle$

$g(x,y,z) = x^2 + y^2 + z^2,\ \nabla g = \langle 2x, 2y, 2z \rangle$

$\lambda y = 2x,\ \lambda x = 2y,\ 2\lambda = 2z,\ \lambda = z$

$y = \frac{2x}{\lambda},\ x = \frac{2(2x)}{\lambda^2},\ \lambda^2 = 4,\ \lambda = \pm 2 = z$

$x^2 + x^2 + 4 = 36,\ 2x^2 = 32,\ x^2 = 16,\ x = \pm 4$

$y = \frac{2x}{\lambda} = \pm\frac{2(4)}{2} = \pm 4$

$max = (4)(4) + 2(2) = 20,\ min = (-4)(4) - 2(2) = -20$

15) $f(x,y,z) = xy + xz,\ \nabla f = \langle y+z, x, x \rangle$

$g(x,y,z) = x^2 + y^2 + z^2,\ \nabla g = \langle 2x, 2y, 2z \rangle$

$\lambda(y+z) = 2x,\ \lambda x = 2y = 2z,\ y = z = \frac{\lambda x}{2}$ $\frac{x\sqrt{2}}{2}$

$2x = \lambda(2(\frac{\lambda x}{2})) = \lambda^2 x,\ \lambda^2 = 2,\ \lambda = \pm\sqrt{2}$

$x^2 + 2(\frac{x\sqrt{2}}{2})^2 = 4,\ 2x^2 = 4,\ x = \pm\sqrt{2},\ y = z = \pm 1$

$max = \sqrt{2} + \sqrt{2} = 2\sqrt{2},\ min = -\sqrt{2} - \sqrt{2} = -2\sqrt{2}$