14.8 Homework

5. $f(x,y) = x^2 + y^2$ $\quad g(x,y) = 2x + 3y$

$\nabla f = \langle f_x, f_y \rangle = \langle 2x, 2y \rangle$
$\nabla g = \langle g_x, g_y \rangle = \langle 2, 3 \rangle$

$\nabla f = \lambda \nabla g \quad \langle 2x, 2y \rangle = \lambda \langle 2, 3 \rangle$

$2x = 2\lambda \quad x = \lambda$
$2y = 3\lambda \quad y = 3/2\,\lambda$

$2x + 3y = 6 \quad 2\lambda + 9/2\,\lambda = 6$
$13/2\,\lambda = 6$
$\lambda = 12/13$
$x = 12/13 \quad y = 36/26 = 18/13$

$(12/13)^2 + (18/13)^2 = 468/169$,

7. $f(x,y) = xy \quad g(x,y) = 4x^2 + 9y^2$

$\nabla f = \langle f_x, f_y \rangle = \langle y, x \rangle$
$\nabla g = \langle g_x, g_y \rangle = \langle 8x, 18y \rangle$

$\nabla f = \lambda \nabla g \quad \langle y, x \rangle = \lambda \langle 8x, 18y \rangle$

$y = \lambda 8x \quad xy = \lambda^2 (144)\, xy$
$x = \lambda 18y \quad 1/144 = \lambda^2$

$\lambda = \pm 1/12$

$\lambda = \frac{1}{12}$

$y = \lambda 8y$

$y = 2/3x$

$4x^2 + 9(2/3x)^2 = 32$

$4x^2 + 4x^2 = 32$

$8x^2 = 32$

$x^2 = 4$

$x = \pm 2$

$y = \pm 4/3$

$\lambda = -1/12$

$y = \lambda 8y$

$y = -2/3x$

$4x^2 + 9(-2/3x)^2 = 32$

$4x^2 + 4x^2 = 32$

$8x^2 = 32$

$x^2 = 4$

$x = \pm 2$

$y = \pm 4/3$

Points: $P(2, 4/3)$ $Q(-2, 4/3)$

$R(-2, -4/3)$, $S(2, -4/3)$

$f(x,y) = xy$

$f(P) = 8/3$

$f(R) = 8/3$

$f(Q) = -8/3$

$f(S) = -8/3$

Min: $8/3$

Max: $-8/3$

9. $f(x,y) = x^2+y^2$ $\quad g(x,y) = x^4+y^4$

$\nabla f = \langle f_x, f_y \rangle = \langle 2x, 2y \rangle$ $\quad \nabla g = \langle g_x, g_y \rangle = \langle 4x^3, 4y^3 \rangle$

$\nabla f = \lambda \nabla g$ $\quad \langle 2x, 2y \rangle = \lambda \langle 4x^3, 4y^3 \rangle$

$2x = \lambda 4x^3$ $\quad x = \lambda 2x^3$ $\quad \lambda = \frac{1}{2x^2}$

$2y = \lambda 4y^3$ $\quad y = \lambda 2y^3$ $\quad \lambda = \frac{1}{2y^2}$

$\frac{1}{2y^2} = \frac{1}{2x^2}$ $\quad 2y^2 = 2x^2$ $\quad x^2 = y^2$ $\quad \pm x = \pm y$

$x^4 + y^4 = 1$

$2x^4 = 1$ $\quad x = \pm \frac{1}{2^{1/4}}, 0, 1$ $\quad y = \pm \frac{1}{2^{1/4}}, 0, 1$

Points: $P(\pm 1/2^{1/4}, \pm 1/2^{1/4})$, $Q(1, 0)$

$R(0, 1)$

$f(x,y) = x^2+y^2$

$f(P) = \sqrt{2}$ $\quad f(Q) = 1$

~~$f(R) = 1$~~

Max: $\sqrt{2}$ $\quad$ Min: 1

11. $f(x,y,z) = 3x+2y+4z$ $\quad g(x,y,z) = 2x^2+2y^2+6z^2$

$\nabla f = \langle 3, 2, 4 \rangle$ $\quad \nabla g = \langle 2x, 4y, 12z \rangle$

$\nabla f = \lambda \nabla g$ $\quad \langle 3, 2, 4 \rangle = \lambda \langle 2x, 4y, 12z \rangle$

$3 = \lambda 2x$ $\quad x = 3/2\lambda$

$2 = \lambda 4y$ $\quad y = 1/2\lambda$

$4 = \lambda 12z$ $\quad z = 1/3\lambda$

$x^2 + 2y^2 + 6z^2 = 1$

$\left(\frac{3}{2\lambda}\right)^2 + 2\left(\frac{1}{2\lambda}\right)^2 + 6\left(\frac{1}{3\lambda}\right)^2 = 1$

$\frac{9}{4\lambda^2} + \frac{1}{2\lambda^2} + \frac{2}{3\lambda^2} = 1$

$\frac{1}{\lambda^2}(41/12) = 1 \qquad \lambda^2 = 41/12 \qquad \lambda = \pm\sqrt{41/12}$

$x = \frac{3}{2\lambda} = \frac{3}{2}\sqrt{12/41} = \pm\sqrt{27/41}$

$y = \frac{1}{2\lambda} = \frac{1}{2}\sqrt{12/41} = \pm\sqrt{3/41}$

$z = \frac{1}{3\lambda} = \frac{1}{3}\sqrt{12/41} = \pm 2\sqrt{123}$

$f(x,y,z) = 3x + 2y + 4z$

Points: $P(\sqrt{27/41}, \sqrt{3/41}, 2\sqrt{123})$

$Q(-P)$

$f(P) = 3.6468$ max

$f(Q) = -3.6468$ min

13. $f(x,y,z) = xy + 2z$ $\quad g(x,y,z) = x^2 + y^2 + z^2$

$\nabla f = \langle y, x, 2 \rangle$ $\quad \nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g$ $\quad \langle y, x, 2 \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$y = \lambda 2x$ $\quad \frac{y}{2x} = \frac{x}{2y}$ $\quad \pm x = y$

$x = \lambda 2y$

$2 = \lambda 2z$ $\quad \frac{1}{2x} = \lambda$ $\quad \lambda = \frac{1}{2}$

$z = \frac{1}{\lambda}$ $\quad z = 2$

$2x^2 + y^2 = 36$

$x^2 = 16$ $\quad x = \pm 4$ $\quad y = \pm 4$ $\quad z = 2$

$f(x,y,z) = xy + 2z$

Points: $P(4,4,2)$ $\quad Q(-4,-4,2)$

| | | |
|---|---|---|
| $f(P) = 20$ | max | |
| $f(Q) = -20$ | min | |

15. $f(x,y,z) = xy + xz$ $\quad g(x,y,z) = x^2 + y^2 + z^2$

$\nabla f = \langle y+z, x, x \rangle$ $\quad \nabla g = \langle 2x, 2y, 2z \rangle$

$\nabla f = \lambda \nabla g$ $\quad \langle y+z, x, x \rangle = \lambda \langle 2x, 2y, 2z \rangle$

$y + z = \lambda 2x$ $\quad 2y = \lambda 2x$

$x = \lambda 2y$ $\quad y = \lambda x$ $\quad y = \pm \frac{x}{\sqrt{2}}$

$x = \lambda 2z$ $\quad y = z$ $\quad z = \lambda x$ $\quad z = \pm \frac{x}{\sqrt{2}}$

$x = \lambda 2 \lambda x$ $\quad x^2 + y^2 + z^2 = 4$

$\frac{1}{2} = \lambda^2$ $\quad x^2 + \frac{x^2}{2} + \frac{x^2}{2} = 4$ $\quad y = \pm 1$

$\lambda = \pm \frac{1}{\sqrt{2}}$ $\quad 2x^2 = 4$ $\quad z = \pm 1$

$x = 2 \pm \sqrt{2}$

$x = \pm\sqrt{2}$ $y = \pm 1$ $z = \pm 1$

Points: $P(\sqrt{2}, 1, 1)$ Don't use other points as these will yield min/max

$Q(-\sqrt{2}, 1, 1)$

$f(P) = 2\sqrt{2}$ max

$f(Q) = -2\sqrt{2}$ min