S1. Write out Lagrange eqs.
S2. Solve for λ in terms of x & y
S3. Solve for x & y using constraint
S4. Calc. f at crit. pt.

14.8 HW

10/18/20

#'s 5, 7, 9, 11, 13, 15 : Find the min. & max. vals. of the fn. (given constraint)

5. $f(x,y) = x^2+y^2$, $2x+3y=6 \Rightarrow y = -2/3\,x + 2$

S1 $\nabla f = \lambda \nabla g$
$\langle 2x, 2y\rangle = \lambda\langle 2,3\rangle \Rightarrow$ $2x = \lambda\cdot 2$, $2y = \lambda \cdot 3$

S2. $\lambda = x$ & $\lambda = 2/3\,y$

S3. $x = 2/3\,y \Rightarrow y = 3/2\,x \Rightarrow 2x + 3\cdot(3/2\,x) = 6 \Rightarrow$
$13x = 12 \Rightarrow x = 12/13$, $y = 3/2 \cdot 12/13 = 18/13$
Crit. pt.: $(12/13, 18/13)$

S4. $f(12/13, 18/13) = (12/13)^2 + (18/13)^2 = 468/169 \approx 2.77$
Constraint: as $|x| \to +\infty$ then so does $|y|$, & hence x^2+y^2 is incr. w/o bound $\Rightarrow$ $468/169$ is a min.

7. $f(x,y) = xy$, $4x^2+9y^2=32$

S1. $\nabla f = \lambda\nabla g$
$\langle y, x\rangle = \lambda\langle 8x, 18y\rangle \Rightarrow$ $y = \lambda(8x)$, $x = \lambda(18y)$

S2. $\lambda = y/8x$ & $\lambda = x/18y$

S3. $y/8x = x/18y \Rightarrow 18y^2 = 8x^2 \Rightarrow y = \pm 2/3\,x$
$4x^2 + 9\cdot(\pm 2/3\,x)^2 = 32 \Rightarrow 4x^2 + 9 \cdot 4x^2/9 = 32 \Rightarrow$
$8x^2 = 32 \Rightarrow x = -2, 2$
$y = 2/3\cdot(-2) = -4/3$, $y = -2/3\cdot(-2) = 4/3$, $y = 2/3 \cdot 2 = 4/3$,
$y = -2/3\cdot 2 = -4/3 \Rightarrow$ Crit. pts.: $(-2,-4/3), (-2,4/3), (2,4/3), (2,-4/3)$

S4. $f(-2,-4/3) = f(2,4/3) = 8/3$; $f(-2,4/3) = f(2,-4/3) = -8/3 \Rightarrow$
$8/3$ is max. val. & $-8/3$ is min. val.

9. $f(x,y) = x^2+y^2$, $x^4+y^4 = 1$

S1. $\nabla f = \lambda\nabla g$
$\langle 2x, 2y\rangle = \lambda\langle 4x^3, 4y^3\rangle \Rightarrow$ $2x = \lambda(4x^3)$, $2y = \lambda(4y^3)$ $\Rightarrow$ $x = 2\lambda x^3$, $y = 2\lambda y^3$

S2. $\lambda = 1/2x^2$ & $\lambda = 1/2y^2$

S3. $1/2x^2 = 1/2y^2 \Rightarrow y^2 = x^2 \Rightarrow y = \pm x$
$x^4 + (\pm x)^4 = 1 \Rightarrow 2x^4 = 1 \Rightarrow x^4 = 1/2 \Rightarrow x = 1/2^{1/4}, -1/2^{1/4}$
Crit. pts.: $(1/2^{1/4}, 1/2^{1/4}), (1/2^{1/4}, -1/2^{1/4}), (-1/2^{1/4}, 1/2^{1/4}), (-1/2^{1/4}, -1/2^{1/4})$
Case 1 : $x = 0 \Rightarrow$ crit. pts.: $(0,-1), (0,1)$
Case 2 : $y = 0 \Rightarrow$ crit. pts.: $(-1,0), (1,0)$
8 CRIT. PTS!!

S4. $f(A_1) = f(A_2) = f(A_3) = f(A_4) = (1/2^{1/4})^2 + (1/2^{1/4})^2 = 2/2^{1/2} = \sqrt{2}$ (max.)

$f(A_5) = f(A_6) = f(A_7) = f(A_8) = 1$ (min.)

11. $f(x,y,z) = 3x + 2y + 4z$, $x^2 + 2y^2 + 6z^2 = 1$

S1. $\nabla f = \lambda \nabla g$
$\langle 3,2,4 \rangle = \lambda \langle 2x, 4y, 12z \rangle$ $\Rightarrow$ $3 = \lambda(2x)$, $2 = \lambda(4y)$, $4 = \lambda(12z)$ $\Rightarrow$ $3/2 = \lambda x$, $1/2 = \lambda x$, $1/3 = \lambda z$

S2. $\lambda = 3/2x$, $\lambda = 1/2y$, $\lambda = 1/3z$

S3. $3/2x = 1/2y = 1/3z \Rightarrow x = 9/2\, z$, $y = 3/2\, z$

$(9/2\, z)^2 + 2(3/2\, z)^2 + 6z^2 = 1 \Rightarrow 123/4\ z^2 = 1 \Rightarrow$

$z_1 = 2/\sqrt{123}$, $z_2 = -2/\sqrt{123}$

$x_1 = 9/2 \cdot 2/\sqrt{123} = 9/\sqrt{123}$, $y_1 = 3/2 \cdot 2/\sqrt{123} = 3/\sqrt{123}$

$x_2 = 9/2 \cdot -2/\sqrt{123} = -9/\sqrt{123}$, $y_2 = 3/2 \cdot -2/\sqrt{123} = -3/\sqrt{123}$

Crit. pts.: $(9/\sqrt{123}, 3/\sqrt{123}, 2/\sqrt{123}) = P_1$ & $P_2 = (-9/\sqrt{123}, -3/\sqrt{123}, -2/\sqrt{123})$

S4. $f(P_1) = 27/\sqrt{123} + 6/\sqrt{123} + 8/\sqrt{123} = 41/\sqrt{123} = \sqrt{41/3} \approx 3.7$ (max.)

$f(P_2) = -27/\sqrt{123} - 6/\sqrt{123} - 8/\sqrt{123} = -41/\sqrt{123} = -\sqrt{41/3} \approx -3.7$ (min.)

13. $f(x,y,z) = xy + 2z$, $x^2 + y^2 + z^2 = 36$

S1. $\nabla f = \lambda \nabla g$
$\langle y, x, 2 \rangle = \lambda \langle 2x, 2y, 2z \rangle$ $\Rightarrow$ $y = 2\lambda x$, $x = 2\lambda y$, $2 = 2\lambda z$

S2. $\lambda = y/2x$, $\lambda = x/2y$, $\lambda = 1/z$

S3. $y/2x = x/2y = 1/z \Rightarrow x^2 = y^2$, $z = 2x/y \Rightarrow x = \pm y$, $z = 2x/y$

$x^2 + y^2 + z^2 - 36 = 0 \Rightarrow 2x^2 - 36 = 0 \Rightarrow x = \pm 4 \Rightarrow y = \pm 4$

8 Crit. pts.: $(\pm 4, \pm 4, \pm 2)$

S4.

$f(0,0,6) = 12$ | $f(0,0,-6) = -12$

$$\left.\begin{array}{l} f(\underline{4,4,2}) = 20 \\ f(-4,4,2) = -12 \\ f(4,-4,2) = -12 \\ f(\underline{-4,-4,2}) = 20 \end{array}\right\} \text{max.} \qquad \begin{array}{l} f(4,4,-2) = 12 \\ \left.\begin{array}{l} f(\underline{-4,4,-2}) = -20 \\ f(\underline{4,-4,-2}) = -20 \end{array}\right\} \text{min.} \\ f(-4,-4,-2) = 12 \end{array}$$

15. $f(x,y,z) = xy + xz$, $x^2 + y^2 + z^2 = 4$

S1. $\nabla f = \lambda \nabla g$
$\langle y+z, x, x \rangle = \lambda \langle 2x, 2y, 2z \rangle$ $\Rightarrow$ $y+z = 2\lambda x$, $x = 2\lambda y$, $x = 2\lambda z$

S2. $\lambda = (y+z)/2x$, $\lambda = x/2y$, $\lambda = x/2z$

S3. $(y+z)/2x = x/2y = x/2z$

14.8 HW Cont.

$$x^2 + \left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{x}{\sqrt{2}}\right)^2 - 4 = 0 \Rightarrow 2x^2 = 4 \Rightarrow x = \pm\sqrt{2}$$

$$\Rightarrow y = z = \pm 1$$

4 crit. pts.: $(\sqrt{2}, 1, 1)$, $(\sqrt{2}, -1, -1)$, $(-\sqrt{2}, 1, 1)$, $(-\sqrt{2}, -1, -1)$

54. $f(\sqrt{2}, 1, 1) = 2\sqrt{2}$ max. $f(\sqrt{2}, -1, -1) = -2\sqrt{2}$ min.

$f(-\sqrt{2}, 1, 1) = -2\sqrt{2}$ min. $f(-\sqrt{2}, -1, -1) = 2\sqrt{2}$ max.