Ch. 14.6 HW # 1, 3, 5, 7, 15, 17, 23, 27, 29, 31 due 10/11

1.) a. $f_x(x,y,z) = 2xy^3 = \frac{df}{dx}$ $f_y(x,y,z) = 3x^2y^2 = \frac{df}{dy}$ $f_z(x,y,z) = 4z^3 = \frac{df}{dz}$

b. $dx/ds = 2s$ $dy/ds = t^2$ $dz/ds = 2st$

c. $\frac{df}{ds} = \frac{df}{dx}\cdot\frac{dx}{ds} + \frac{df}{dy}\cdot\frac{dy}{ds} + \frac{df}{dz}\cdot\frac{dz}{ds} = 2\cancel{x}\cancel{y}^3\cdot 2s + 3\cancel{x}^2\cancel{y}^2\cdot t^2 + 4\cancel{z}^3\cdot 2st$
$= 4s^6t^6 + 3s^6t^6 + 8s^7t^4 = 7s^6t^6 + 8s^7t^4$

3.) $\frac{df}{ds}, \frac{df}{dr}$; $f(x,y,z) = xy + z^2$, $x = s^2$, $y = 2rs$, $z = r^2$

$\frac{df}{dx} = y$, $\frac{df}{dy} = x$, $\frac{df}{dz} = 2z$, $\frac{dx}{ds} = 2s$, $\frac{dy}{ds} = 2r$, $\frac{dz}{ds} = 0$, $\frac{dx}{dr} = 0$, $\frac{dy}{dr} = 2s$, $\frac{dz}{dr} = 2r$

$\frac{df}{ds} = \frac{df}{dx}\cdot\frac{dx}{ds} + \frac{df}{dy}\cdot\frac{dy}{ds} + \frac{df}{dz}\cdot\frac{dz}{ds} = \cancel{y}\cdot 2s + \cancel{x}\cdot 2r + 2\cancel{z}\cdot 0 = 4rs^2 + 2rs^2 = 6rs^2$

$\frac{df}{dr} = \frac{df}{dx}\cdot\frac{dx}{dr} + \frac{df}{dy}\cdot\frac{dy}{dr} + \frac{df}{dz}\cdot\frac{dz}{dr} = \cancel{y}\cdot 0 + \cancel{x}\cdot 2s + 2\cancel{z}\cdot 2r = 2s^3 + 4r^3$

5.) $\frac{dg}{du}, \frac{dg}{dv}$; $g(x,y) = \cos(x-y)$, $x = 3u - 5v$, $y = -7u + 15v$

$\frac{dg}{du} = -\sin(x-y)\cdot 3 - (-\sin(x-y))(-7) = -3\sin(x-y) - 7\sin(x-y) = -10\sin(x-y) = -10\sin(10u-20v)$

$\frac{dg}{dv} = -\sin(x-y)\cdot(-5) + \sin(x-y)\cdot 15 = 5\sin(x-y) + 15\sin(x-y) = 20\sin(x-y) = 20\sin(10u-20v)$

7.) $\frac{dF}{dy}$; $F(u,v) = e^{u+v}$, $u = x^2$, $v = xy$

$\frac{dF}{dy} = e^{u+v}\cdot 0 + e^{u+v}\cdot x = xe^{u+v} = xe^{x^2+xy}$

15.) dg/du at $(u,v) = (0,1)$, where $g(x,y) = x^2 - y^2$, $x = e^u\cos v$, $y = e^u\sin v$

$dg/du = 2x\cdot e^u\cos v - 2y\cdot e^u\sin v = 2e^u\cos v\, e^u\cos v - 2e^u\sin v\, e^u\sin v = 2e^u(\cos^2 v - \sin^2 v) = 2e^u\cos 2v$

? 17.)

23.) $(df/dx)^2 - (df/dy)^2 = \left(\frac{df}{dx}\frac{dx}{ds} + \frac{df}{dy}\frac{dy}{ds}\right)\left(\frac{df}{dx}\frac{dx}{dr} + \frac{df}{dy}\frac{dy}{dr}\right)$

$\left(\frac{df}{dx} + \frac{df}{dy}\right)\left(\frac{df}{dx} - \frac{df}{dy}\right) = (df/dx + df/dy)(df/dx - df/dy)$ ✓

27.) dz/dx, $x^2y + y^2z + xz^2 - 10 = 0$

$2xy + y^2\, dz/dx + z^2 + 2xz\, dz/dx = 0 \Rightarrow \frac{dz}{dx} = \frac{-2xy - z^2}{y^2 + 2xz}$

29.) dz/dy, $e^{xy} + \sin(xz) + y = 0$

$xe^{xy} + x\cos(xz)\frac{dz}{dy} + 1 = 0 \Rightarrow \frac{dz}{dy} = \frac{-1 - xe^{xy}}{x\cos(xz)}$

31.) dw/dy, $\frac{1}{w^2 + x^2} + \frac{1}{w^2 + y^2} - 1 = 0$ at $(x,y,w) = (1,1,1)$

$-\frac{2w}{(w^2+x^2)^2}\frac{dw}{dy} - \frac{2w}{(w^2+y^2)^2}\frac{dw}{dy} + \frac{2y}{(w^2+y^2)^2} = 0$

$\frac{dw}{dy} = -\frac{2y}{(w^2+y^2)^2}\cdot 1\Big/\left(-\frac{2w}{(w^2+x^2)^2} - \frac{2w}{(w^2+y^2)^2}\right)$

Ch. 14.7 HW # 1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35 due 10/11/2020

1.) a. $f_x(x,y) = 2x - 4y = 0 \rightarrow 2a - 4b = 0 \rightarrow a = 2b$ ✓ $\frac{2^{3/4}}{9} = 2^{3/2}$ ✓

$f_y(x,y) = 4y^3 - 4x = 0 \rightarrow y^3 = x$: $P(0,0) \rightarrow 0^3 = 0$, $P(\sqrt{2}\cdot 2, \sqrt{2}) \rightarrow 2^{3/2} = \sqrt{8} = 2^{3/2} = \sqrt{2}$

$P(-2^{3/2}, -\sqrt{2}) \rightarrow (-\sqrt{2})^3 = -2^{3/2} \rightarrow -2^{3/2} = -2^{3/2}$ ✓

b. $f_{xx}(x,y) = 2$, $f_{yy}(x,y) = 12y^2$, $f_{xy}(x,y) = -4$ Critical points: $(0,0)$

$D = (2)(12y^2) - (-4)^2 = 24y^2 - 16 = -16 \rightarrow f(0,0)$ is a saddle point

3.) $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_x(x,y) = 2x + y = 0 \rightarrow y = -2x$ $f_y(x,y) = 32y^4 + x - 6y - 3y^2 = 0$ critical points: $(0,0)$

$(0,0)$ is a local minimum

5.) $f(x,y) = y^2x - yx^2 + xy$

a. $f_x(x,y) = y^2 - 2yx + y = y(y - 2x + 1) = 0$ ✓ $f_y(x,y) = 2yx - x^2 + x = x(2y - x + 1) = 0$ ✓

b. $y^2 + x^2 - 4yx + y - x = 0$

critical Points: $(0,0)$, $(0,-1)$, $(1,0)$, $(1,3)$

c. $f_{xx}(x,y) = -2y$ $f_{yy}(x,y) = 2x$ $f_{xy}(x,y) = 2y - 2x + 1$

$$\left.\begin{aligned} &(0,0) \rightarrow D = 0\cdot 0 - (1)^2 = -1 \\ &(0,-1) \rightarrow D = 2\cdot 0 - (-1)^2 = -1 \\ &(1,0) \rightarrow D = 0\cdot 2 - (-1)^2 = -1 \\ &(1,3) \rightarrow D = -6\cdot 2 - (5)^2 = -37 \end{aligned}\right\} \text{All are saddle points b/c } D < 0$$

7.) $f(x,y) = x^2 + y^2 - xy + x$ $f_x(x,y) = 2x - y + 1 = 0$, $f_y(x,y) = 2y - x = 0$

$2x - 2y - y + x + 1 = 3x - 3y + 1 = 0$ critical points: $(0, \frac{1}{3})$ $(-\frac{1}{3}, 0)$

$f_{xx}(x,y) = 2$ $f_{yy}(x,y) = 2$ $f_{xy}(x,y) = -1$

$(0, \frac{1}{3})$ and $(-\frac{1}{3}, 0) \rightarrow D = 2\cdot 2 - (-1)^2 = 3 \rightarrow$ Both are local mins

11.) $f(x,y) = 4x - 3x^3 - 2xy^2$ $f_x(x,y) = 4 - 9x^2 - 2y^2$ $f_y(x,y) = -4xy$

Critical points: $(0, \sqrt{2})$, $(\frac{2}{3}, 0)$

$f_{xx}(x,y) = -18x$ $f_{yy}(x,y) = -4x$ $f_{xy}(x,y) = -4y$

$(0, \sqrt{2}) \rightarrow D = 0\cdot 0 - (-4\sqrt{2})^2 = -32 \leftarrow$ saddle point b/c $D < 0$

$(\frac{2}{3}, 0) \rightarrow D = (-12)(-\frac{8}{3}) - (0)^2 = 32 \leftarrow$ local max b/c $D > 0$ and f_{xx} and $f_{yy} < 0$

13.) $f(x,y) = x^4 + y^4 - 4xy$ $f_x(x,y) = 4x^3 - 4y$ $f_y(x,y) = 4y^3 - 4x$

Critical points = $(0,0)$, $(1,1)$, $(-1,-1)$

$f_{xx}(x,y) = 12x^2$ $f_{yy}(x,y) = 12y^2$ $f_{xy}(x,y) = -4$

$(0,0) \rightarrow D = 0\cdot 0 - (-4)^2 = -16 \leftarrow$ Saddle point b/c $D < 0$

$(1,1) \rightarrow D = 12\cdot 12 - (-4)^2 = 128 \leftarrow$ local min b/c $D > 0$ and $f_{xx}, f_{yy} > 0$

$(-1,-1) \rightarrow D = 12\cdot 12 - (-4)^2 = 128 \leftarrow$ local max b/c $D > 0$ and $f_{xx}, f_{yy} < 0$

19.) $f(x,y) = \ln x + 2\ln y - x - 4y$ $f_x(x,y) = \frac{1}{x} - 1$ $f_y(x,y) = \frac{2}{y} - 4$

Critical point: $(1, \frac{1}{2})$

$f_{xx}(x,y) = -\frac{1}{x^2}$ $f_{yy}(x,y) = -\frac{2}{y^2}$ $f_{xy}(x,y) = 0$

$(1,\frac{1}{2}) \to D = (-1)\cdot(-8) - (0)^2 = 8 \to$ local max b/c $D > 0$ and $f_{xx}, f_{yy} < 0$

21.) $f(x,y) = x - y^2 - \ln(x+y)$ $f_x(x,y) = 1 - \frac{1}{x+y}$ $f_y(x,y) = -2y - \frac{1}{x+y}$

No critical points, 2nd Deriv test inconclusive

23.) $f(x,y) = (x+3y)e^{y-x^2}$ $f_x(x,y) = -2xe^{y-x^2}$ $f_y(x,y) = 3e^{y-x^2}$

No critical points b/c $f_y(x,y)$ can never be 0

29.) $f(x,y) = x+y$, $0 \le x \le 1$, $0 \le y \le 1$ $f_x(x,y) = 1$ $f_y(x,y) = 1$

critical point: $(1,1)$ $f(1,1) = 2$

left: $x = 0$ $f(0,y) = y = F(y)$ $F(0) = 0$ $F(1) = 1$

max: 1 min: 0

Right: $x = 1$ $f(1,y) = 1+y$ $F(0) = 1$ $F(1) = 2$

max: 2 min: 1

UP: $y = 0$ $f(x,0) = x = F(x)$ $F(0) = 0$ $F(1) = 1$

~~max~~: Max = 2 min = 0

35.) $f(x,y) = x + y - x^2 - y^2 - xy$ $f_x(x,y) = 1 - 2x - y$ $f_y(x,y) = 1 - 2y - x$

a. $y = 1 - 2x$ $y = \frac{1-x}{2}$ $2 - 4x = 1 - x$ $1 = 3x$ $(\frac{1}{3}, \frac{1}{3})$

$f(\frac{1}{3},\frac{1}{3}) = \frac{1}{3} + \frac{1}{3} - \frac{1}{9} - \frac{1}{9} - \frac{1}{9} = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

b. $f(x,0) = x - x^2 = F(x)$ $F'(x) = 1 - 2x = 0$ $x \to \frac{1}{2}$

$F(\frac{1}{2}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$ $F(0) = 0$ $F(2) = -2$

Max: $\frac{1}{4}$ min: -2

c. $f(x,2) = x + 2 - x^2 - 4 - 2x = -x^2 - x - 2 = F(x)$ $F'(x) = -2x - 1$ $x = -\frac{1}{2}$

$F(-\frac{1}{2}) = -\frac{1}{4} + \frac{1}{2} - 2 = -7/4$ $F(0) = -2$ $F(2) = -8$

max: $-7/4$ min: -8

$f(0,y) = y - y^2 = F(y)$ $\to$ No need to continue, will produce same values

d. max: 1/3 min: -8