14.6

1.

(a) $f(x,y,z)=x^2y^3+z^4$

$x=s^2 \quad y=st^2 \quad z=s^2t$

$\frac{\partial f}{\partial x}=2xy^3 \quad \frac{\partial f}{\partial y}=x^2\cdot 3y^2$

$\frac{\partial f}{\partial z}=4z^3$

(b) $\frac{\partial x}{\partial s}=2s \quad \frac{\partial y}{\partial s}=t^2 \quad \frac{\partial z}{\partial s}=2st$

(c) $\frac{\partial f}{\partial s}=\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial s}+\frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial s}$

$=2xy^3\cdot 2s+3x^2y^2\cdot t^2+4z^3\cdot 2st$

$=4xy^3s+3x^2y^2t^2+8z^3st$

plug in x,y,t

$=7s^6t^6+8s^7t^4$

3. $\frac{\partial f}{\partial x}=y \quad \frac{\partial f}{\partial y}=x \quad \frac{\partial f}{\partial z}=2z$

$\frac{\partial x}{\partial s}=2s \quad \frac{\partial y}{\partial s}=2r \quad \frac{\partial z}{\partial s}=0$.

$\frac{\partial x}{\partial r}=0 \quad \frac{\partial y}{\partial r}=2s \quad \frac{\partial z}{\partial r}=2r$

$\frac{\partial f}{\partial s}=2ys+2xr+0$

$=\cancel{2y}\ 2\cdot 2rs\cdot s+2\cdot s^2\cdot r+0$

$=4rs^2+2rs^2+0$

$=6rs^2$

$\frac{\partial f}{\partial r}=0+2xs+\cancel{\infty}4rz$

$=2\cdot s^2\cdot s+4\cdot r\cdot r^2$

$=2\cancel{4}s^3+4r^3$

5. $\frac{\partial g}{\partial x}=-\sin(x-y)\cdot 1 \quad \frac{\partial g}{\partial y}=-\sin(x-y)\cdot(-1)$

$=\sin(x-y)$

$\frac{\partial x}{\partial u}=\cancel{5}\ 3 \quad \frac{\partial y}{\partial u}=-7$

$\frac{\partial x}{\partial v}=-5 \quad \frac{\partial y}{\partial v}=15$

$\frac{\partial g}{\partial u}=-\sin(x-y)\cdot 3+\sin(x-y)\times(-7)$

$=-3\sin(x-y)\cancel{+}-7\sin(x-y)$

$=-3\sin(3u-5v+7u-15v)-7\sin(3u-5v+7u-15v)$

$=-10\sin(10u-20v)$

$\frac{\partial g}{\partial v}=5\sin(x-y)+15\sin(x-y)$

$=20\sin(10u-20v)$

7. $\frac{\partial F}{\partial u}=e^{u+v}\cdot 1$

$\frac{\partial F}{\partial v}=e^{u+v}\cdot 1$

$\frac{\partial u}{\partial x}=2x \quad \frac{\partial v}{\partial x}=y$

$\frac{\partial u}{\partial y}=0 \quad \frac{\partial v}{\partial y}=x$

$\frac{\partial F}{\partial y}=e^{u+v}\cdot 0+e^{u+v}\cdot x$.

$=e^{u+v}\cdot x$

$=xe^{x^2+xy}$

15. $\frac{\partial g}{\partial x}=2x \quad \frac{\partial g}{\partial y}=-2y$

$\frac{\partial x}{\partial u}=\cancel{2e^u}\ e^u\cos v \quad \frac{\partial y}{\partial u}=e^u\sin v$

$(u,v)=(0,1)$

plug in

$\frac{\partial g}{\partial u}=2x\cdot e^u\cos v\ \cancel{+}-2y\cdot e^u\sin v$

$=2\cdot e^u\cos v\cdot e^u\cos v-2\cdot e^u\sin v\cdot e^u\sin v=1.998781654$

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23. $x = s+t$

$y = s-t$

$\cancel{\frac{\partial^2 f}{\partial x^2}}\ (\frac{\partial f}{\partial x})^2 - (\frac{\partial f}{\partial y})^2 = \frac{\partial f}{\partial s}\frac{\partial f}{\partial t}$

for example: $f(x,y) = x+y$

$\frac{\partial f}{\partial x} = 1 \quad \frac{\partial f}{\partial y} = 1$

$\frac{\partial x}{\partial s} = 1 \quad \cancel{\frac{\partial x}{\partial t}}\ \frac{\partial y}{\partial s} = 1$

$\frac{\partial x}{\partial t} = 1 \quad \frac{\partial y}{\partial t} = -1$

$1^2 - 1^2 = (1 \times 1 + 1 \times 1) \cdot (1 \times 1 - 1 \times 1)$

$0 = 0$

29.

$e^{xy} \cdot x + \cancel{\sin}\overset{\cos}{}(xz) \cdot xz' + 1 = 0.$

$e^{xy} \cdot x + \cancel{\sin}\overset{\cos}{}(xz) \cdot xz' + 1 = 0.$

$\underset{\cos}{\cancel{\sin}}(xz) \cdot xz' = -1 - e^{xy} \cdot x$

$xz' = \frac{-1 - e^{xy} \cdot x}{\cos\cancel{\sin}(xz)}$

$z' = \frac{-1 - \cancel{x}e^{xy}}{\cos\cancel{\sin}(xz)} \times \frac{1}{x}$

$= \frac{-1 - e^{xy}}{x\,\underset{\cos}{\cancel{\sin}}(xz)}$

27.

$2xy + y^2 z' + z^2 + x \cdot 2zz' = 0.$

$y^2 z' + 2xzz' = -2xy - z^2$

$z'(y^2 + 2xz) = -2xy - z^2$

$z' = \frac{-2xy - z^2}{y^2 + 2xz}$

31. $\frac{0 - (2w \cdot w')}{(w^2+x^2)^2} + \frac{0 - (2w \cdot w' + 2y)}{(w^2+y^2)^2} = 0$

$\frac{-2ww'}{(w^2+x^2)^2} - \frac{2ww' - 2y}{(w^2+y^2)^2} = 0$

$(x, y, w) = (1, 1, 1)$

$\frac{-2w'}{4} - \frac{2w' - 2}{4} = 0$

$\frac{-2w' - 2w' + 2}{4} = 0.$

$-2w' - 2w' + 2 = 0.$

$-2w' - 2w' = -2$

$w'(-2-2) = -2$

$w' = \frac{-2}{-4} = \frac{1}{2}$

14.7

1.

(a) $f_x(x,y) = 2x - 4y$

$2x - 4y = 0$

$2x = 4y$

$x = 2y$.

$a = 2b$.

$f_y(x,y) = 4y^3 - 4x$

$4y^3 - 4x = 0$.

$4y^3 - 4 \cdot 2y = 0$

$4y^3 - 8y = 0$

$y_1 = \sqrt{2}$ $y_2 = -\sqrt{2}$ $y_3 = 0$.

$x_1 = 2\sqrt{2}$ $x_2 = -2\sqrt{2}$ $y_3 = 0$.

$P = (0,0).\ (2\sqrt{2}, \sqrt{2}),\ (-2\sqrt{2}, -\sqrt{2})$

b.

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 2 \times 12y^2 - (-4)^2$

$D(2\sqrt{2}, \sqrt{2}) = D(-2\sqrt{2}, -\sqrt{2})$

$D = 2 \times 12(\sqrt{2})^2 - 16$

$= 2 \times 12 \times 2 - 16$

$= 2 + 24 - 16 > 0$.

$f_{xx} > 0$

$\therefore (2\sqrt{2}, \sqrt{2})$ and $(-2\sqrt{2}, -\sqrt{2})$ are local minimum.

$D(0,0) = -16 < 0$.

then $(0,0)$ is a saddle point

absolute minimum of f is $(2\sqrt{2})^2 + (\sqrt{2})^4 - 4 \times 2\sqrt{2} \times \sqrt{2}$

$= -4$.

3. $f_x = 2x + y$

$f_y = 32y^3 + x - 6y - 3y^2$

$f_{xx} = 2$

$f_{xy} = 1$

$f_{yy} = 96y^2 - 6 - 6y$.

$2x + y = 0$ $32y^3 + x - 6y - 3y^2 = 0$.

$y_1 = \frac{1}{2}$ $y_2 = -\frac{13}{32}$ $y_3 = 0$

$x_1 = -\frac{1}{4}$ $x_2 = \frac{13}{64}$ $x_3 = 0$.

$D = f_{xx} f_{yy} - f_{xy}^2$

$= 2 \times (-6) - 1 < 0$.

$(0,0)$ is a saddle point.

$D = 2 \times (96 \cdot \frac{1}{4} - 6 - 6 \times \frac{1}{2}) - 1 > 0$.

$(-\frac{1}{4}, \frac{1}{2})$ is the local minimum

$D = 2 \times (96 \cdot (\frac{13}{32})^2 - 6 + 6 \times \frac{13}{32}) - 1 > 0$

$(\frac{13}{64}, -\frac{13}{32})$ is the local minimum

5.

(a) $f_x = y^2 - 2xy + y = 0$

$f_y = 2yx - x^2 + x = 0$

$y(y - 2x + 1) = 0$ $x(2y - x + 1) = 0$

$y^2 - 2xy + y = 0$ $2xy - x^2 + x = 0$.

(b) $x = 0$ $y = 0$. $0^2 - 0 + 0 = 0 \cdot (0,0)$

$x = 0$ $y = -1$ $(0, -1)$

$y = 0$ $x = 1$ $(1, 0)$.

$x = \frac{1}{3}$ $y = -\frac{1}{3}$ $(\frac{1}{3}, -\frac{1}{3})$

(c) $f_{xx} = -2y$ $f_{yy} = 2x$ $f_{xy} = 2y - 2x + 1$

$D = f_{xx} f_{yy} - f_{xy}^2$

$= 0 \times 0 - 1^2 < 0$.

$(0,0)$ is a saddle point.

$= 2 \times 0 - (-2 + 1)^2$

$= 0 - 1 = -1 < 0$.

$(0,-1)$ is a saddle point

$= 0 \times 2 - (0 - 2 + 1)^2$

$= 0 - (-1)^2$

$= -1 < 0$

$(1,0)$ is a saddle point

$= \frac{2}{3} \times \frac{2}{3} - (2 \times (-\frac{1}{3}) - 2 \times \frac{1}{3} + 1)^2$

$= \frac{4}{9} - \frac{1}{9} > 0 \cdot (\frac{1}{3}, -\frac{1}{3})$ is a local minimum.

7. $f_x = 2x - y + 1$

$f_y = 2y - x$

$f_{xx} = 2$

$f_{xy} = -1$

$f_{yy} = 2$

$2x - y + 1 = 0 \quad 2y - x = 0$.

$y = -\frac{1}{3} \quad x = -\frac{2}{3}$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$= 2 \times 2 - (-1)^2$

$= 2 - 1 = 1 > 0$

$f_{xx} > 0$.

$(-\frac{2}{3}, -\frac{1}{3})$ is a local minimum

11. $f_x = 4 - 9x^2 - 2y^2$

$f_y = -2x \cdot 2y = -4xy$

$f_{xx} = -18x$.

$f_{xy} = -4y$

$\cancel{f_{xy}} f_{yy} = -4x$

$4 - 9x^2 - 2y^2 = 0 \quad \cancel{-6}xy = 0$

$x = 0 \quad y = \pm\sqrt{2}$

$y = 0 \quad x = \pm\frac{2}{3}$

$D = f_{xx} f_{yy} - (f_{xy})^2$

$\cancel{= 0 \times (\sqrt{2}) - 0} \quad \sqrt{2} > 0$

$= 0 \times 0 - (4\sqrt{2})^2 < 0$.

$(0, \pm\sqrt{2})$ is a saddle point.

$x = \pm\frac{2}{3} \quad y = 0$.

$D = -18 \times \frac{2}{3} \cdot -4 \cdot \frac{2}{3} - (0)^2$

$= -32 \cdot (-\frac{8}{3}) > 0$. $f_{xx} < 0$.

$(\frac{2}{3}, 0)$ is a local maximum

$D = -18 \times (-\frac{2}{3}) \cdot -4 \cdot (-\frac{2}{3}) - 0^2$

$= 32 \cdot \frac{8}{3} > 0$. $f_{xx} > 0$.

$(-\frac{2}{3}, 0)$ is a local minimum.

13. $f_x = 4x^3 - 4y$

$f_y = 4y^3 - 4x$

$f_{xx} = 12x^2$

$f_{xy} = -4$

$f_{yy} = 12y^2$

$4x^3 - 4y = 0 \quad 4y^3 - 4x = 0$.

$x^3 - y = 0 \quad y^3 - x = 0$.

$y = 0 \quad x = 0$

$y = 1 \quad x = 1$

$y = -1 \quad x = -1$

$D = f_{xx} f_{yy} - f_{xy}^2$

$= 0 \cdot 0 - 16 < 0$.

$(0, 0)$ is a saddle point

$D = 12 \cdot 12 - 16 > 0$.

$f_{xx} > 0$.

$(1, 1)$ is a local minimum

$D = 12 \cdot 12 - 16 > 0 \quad f_{xx} > 0$

$(-1, -1)$ is a local minimum

19. $f_x = \frac{1}{x} - 1$

$f_y = \frac{2}{y} - 4$

$f_{xx} = -x^{-2}$

$f_{xy} = 0$

$f_{yy} = -2y^{-2}$

$\frac{1}{x} - 1 = 0 \quad \frac{2}{y} - 4 = 0$.

$x = 1 \quad y = \frac{1}{2}$

$D = -1 \times (-8) - 0$

$= 8 > 0 \quad f_{xx} < 0$

$(1, \frac{1}{2})$ is a local maximum.

21. $f_x = 1 - \frac{1}{x+y} \cdot 1$

$f_y = -2y - \frac{1}{x+y} \cdot 1$

$f_{xx} = (x+y)^{-2}$

$f_{xy} = (x+y)^{-2}$

$f_{yy} = -2 + (x+y)^{-2}$

$1 - \frac{1}{x+y} = 0 \quad -2y - \frac{1}{x+y} = 0$

$y = -\frac{1}{2} \quad x = \frac{3}{2} \quad (\frac{3}{2}, -\frac{1}{2})$

$D = 1 \times (-1) - 1$

$= -2 < 0$.

$(\frac{3}{2}, -\frac{1}{2})$ is a saddle point

23. $f_x = 1\cdot e^{y-x^2} + (x+3y)\cdot e^{y-x^2}\cdot(-2x)$

$f_y = 3e^{y-x^2} + (x+3y)\cdot e^{y-x^2}\cdot 1$

$f_{xx} = e^{y-x^2}\cdot(-2x) + (-6x-6y)e^{y-x^2} + (-2x^2-6xy)\cdot e^{y-x^2}\cdot(-2x)$

$f_{xy} = e^{y-x^2}\cdot 1 + (-6x)e^{y-x^2} + (-2x^2-6xy)\cdot e^{y-x^2}\cdot 1$

$f_{yy} = 3e^{y-x^2}\cdot 1 + 3e^{y-x^2} + (x+3y)\cdot e^{y-x^2}\times 1$

$e^{y-x^2} + (-2x^2-6xy)\cdot e^{y-x^2} = 0.$ $\quad 3e^{y-x^2} + (x+3y)\cdot e^{y-x^2} = 0.$

$x = -\frac{1}{6}$ $\quad y = -\frac{17}{18}$

$D = f_{xx}f_{yy} - (f_{xy})^2$

$= e^{-\frac{17}{18}-\frac{1}{6}^2}\cdot(-2\times-\frac{1}{6}) + \ldots\ \cdot(-2\times-\frac{1}{6})\cdot 3e^{-\frac{17}{18}-(\frac{1}{6})^2}\ldots\times 1 - \ldots > 0$

$f_{xx} > 0$ $\therefore (-\frac{1}{6}, -\frac{17}{18})$ is a local minimum.

29. $f(x,y) = x+y$

$f_x = 1$ $\quad f_y = 1$ $\quad 0\le x\le 1$, $\quad 0\le y\le 1$

on the left side

$x=0$ $\quad 0\le y\le 1$

$f(0,y) = y = F(y)$

$F'(y) = 1$

$F(0) = 0$

$F(1) = 1$

abs min 0

abs max 1

on the right side

$x=1$ $\quad 0\le y\le 1$

$f(1,y) = 1+y = F(y)$

$F'(y) = 1$

$F(0) = 1$

$F(1) = 2$

abs min = 1

abs max = 2.

on the down side

$y=0$ $\quad 0\le x\le 1$

$f(x,0) = x = F(x)$

$F'(x) = 1$

$F(0) = 0$

$F(1) = 1$

abs min is 0

abs max is 1

on the up side

$y=1$ $\quad 0\le x\le 1$

$f(x,1) = x+1$

$F'(x) = 1$

$F(0) = 1$

$F(1) = 2$

abs min = 1

abs max = 2.

For abs min 0, 1, 0, 1 so abs min is 0.

For abs max 1, 2, 1, 2 so abs max is 2

35. $f_x = 1 - 2x - y$ $\quad 0\le x\le 2$

$f_y = 1 - 2y - x$ $\quad 0\le y\le 2$

~~on the left side~~

(a) $1-2x-y=0$ $\quad 1-2y-x=0.$

$x = \frac{1}{3}$ $\quad y = \frac{2}{3}$ $\quad (-\frac{1}{3}, \frac{2}{3})$

$f = -\frac{1}{3} + \frac{2}{3} - \frac{1}{9} - \frac{4}{9} + \frac{2}{9} = 0.$

(b) on the bottom edge.

$y=0$ $\quad 0\le x\le 2$

$f(x,0) = x - x^2 = F(x)$

$F'(x) = 1 - 2x$ $\quad x = \frac{1}{2}$ $\quad F'(x) = 0$

$F(\frac{1}{2}) = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$F(0) = 0$

$F(2) = 2 - 4 = -2.$

abs min is -2

abs max is $\frac{1}{4}$.

on the left side

$x=0 \quad 0\le y\le 2$

$f(0,y)=y-y^2=F(y)$

$F'(y)=1-2y. \quad y=\frac{1}{2} \quad F'(y)=0$

$F(\frac{1}{2})=\frac{1}{2}-\frac{1}{4}=\frac{1}{4}$

$F(0)=0. \quad F(2)=2-4=-2$

abs min is -2,

abs max is $\frac{1}{4}$.

on the right side.

$x=2 \quad 0\le y\le 2$

$f(2,y)=2+y-4-y^2-2y=F(y)$

$F'(y)=1-2y-2 \quad y=-\frac{1}{2} \quad F'(y)=0$

$F(-\frac{1}{2})=2-\frac{1}{2}-4-\frac{1}{4}+1=-\frac{7}{4}$

$F(0)=2+0-4-0-0=-2$

$F(2)=2+2-4-4-4=-8.$

abs min $=-8$

abs max $=-\frac{7}{4}$

on the up side

$y=2 \quad 0\le x\le 2$

$f(x,2)=x+2-x^2-4-2x=F(y)$

$F'(y)=1-2x-2 \quad x=-\frac{1}{2} \quad F'(y)=0$

$F(-\frac{1}{2})=-\frac{1}{2}+2-\frac{1}{4}-4+1=-\frac{7}{4}$

$F(0)=0+2-0-4-0=-2$

$F(2)=2+2-4-4-4=-8$

abs min $=-8$

abs max $=-\frac{7}{4}$.

For abs max $\frac{1}{4}$, $\frac{1}{4}$, $-\frac{7}{4}$, $-\frac{7}{4}$, 0.

so the abs max is $\frac{1}{4}$

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