Math 251 Shaun Goda Section 23 H/W #5

14.6:

1) (a) $f_x(x,y,z) = 2xy^3$, $f_y(x,y,z) = 3y^2z^2$, $f_z(x,y,z) = \boxed{4z^3}$

(b) $\frac{\partial x}{\partial s} = 2s$ $\frac{\partial y}{\partial s} = t^2$ $\frac{\partial z}{\partial s} = \boxed{2st}$

(c) $\frac{\partial f}{\partial s} = (2xy^3)(2s) + (3y^2z^2)(t^2) + (4z^3)(2st)$

$= 4sxy^3 + 3ty^2x^2 + 8stz^3 = \boxed{7s^6t^6 + 8s^7t^4}$

3) $f_x(x,y,z) = y$, $f_y(x,y,z) = x$, $f_z(x,y,z) = 2z$

$\frac{\partial x}{\partial s} = 2s$ $\frac{\partial y}{\partial s} = 2r$ $\frac{\partial z}{ds} = 0$ $4rs^2 + 2rs^2 = \boxed{6rs^2}$

$\frac{\partial x}{\partial r} = 0$ $\frac{\partial y}{\partial r} = 2s$ $\frac{\partial z}{\partial r} = 2r$ ↑ $\boxed{2s^3 + 4r^3}$

$\frac{\partial f}{\partial s} = (y)(2s) + (x)(2r) + (2z)(0) = 2sy + 2rx$ ↑

$\frac{\partial f}{\partial r} = (y)(0) + (x)(2s) + (2z)(2r) = 2sx + 4rz$

5) $g_x(x,y) = -\sin(x-y)(1)$ $g_y(x,y) = -\sin(x-y)(-1)$

$\frac{\partial x}{\partial u} = 3$ $\frac{\partial y}{\partial u} = -7$, $\frac{\partial x}{\partial v} = -5$ $\frac{\partial y}{\partial v} = 15$

$\frac{\partial g}{\partial u} = (-\sin(x-y))(3) + (\sin(x-y))(-7) = -10\sin(x-y)$

$\frac{\partial g}{\partial v} = (-\sin(x-y))(-5) + (\sin(x-y))(15) = 20\sin(x-y)$

$\frac{\partial g}{\partial u} = -10\sin(x-y) = -10\sin((3u-5v)-(-7u+15v))$
$= \boxed{-10\sin(10u-20v)}$

$\frac{\partial g}{\partial v} = 20\sin(x-y) = -10\sin((3u-5v)-(-7u+15v))$

$= \boxed{20\sin(10u-20v)}$

7) $F_u(u,v) = e^{u+v}$ $F_v(u,v) = e^{u+v}$

$\frac{\partial u}{\partial x} = 2x$ $\frac{\partial u}{\partial y} = 0$ $\frac{\partial v}{\partial x} = y$ $\frac{\partial v}{\partial y} = x$

$$\frac{\partial F}{\partial y} = (e^{u+v})(0) + (e^{u+v})(x) = xe^{u+v}$$

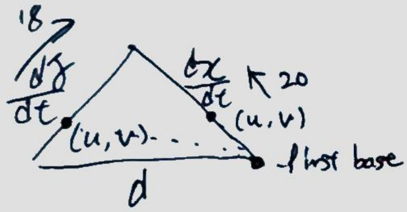
$$= \boxed{xe^{x^2+xy}}$$

15) $g_x(x,y) = 2x$ $g_y(x,y) = -2y$ $\frac{\partial x}{\partial u} = e^u$ $\frac{\partial y}{\partial u} = \sin v\, e^u$

$\frac{\partial x}{\partial v} = 0$ $\frac{\partial y}{\partial v} = e^u \cos v$

$$\frac{\partial g}{\partial u} = (2x)(e^u) + (-2y)(\sin v e^u) = 2e^{2u}\cos v - 2e^{2u}\sin^2 v$$

@ $(u,v) = (0,1)$, $\frac{\partial g}{\partial u} = \boxed{2\cos(1) - 2\sin^2(1) \approx 1.999}$

17) 

$d^2 = (y')^2 + (x')^2$

$x = -20t$, $y = -18t$

$\frac{\partial x}{\partial t} = -20$ $\frac{\partial y}{\partial t} = -18$

$f_x(x,y) = 2x$ $f_y(x,y) = 2y$

$$\frac{\partial d}{\partial t} = (2x)(-20) + (2y)(-18) = -40x - 36y$$

$$\left.\frac{\partial d}{\partial t}\right|_{(8,6)} = -40(8) - 36(6) = \boxed{-536}$$

I don't think it's correct

I do not know how to solve this question...

23) $\frac{\partial x}{\partial s} = 1 \quad \frac{\partial y}{\partial s} = 1 \quad \frac{\partial x}{\partial t} = 1 \quad \frac{\partial y}{\partial t} = -1$

$$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}(1) + \frac{\partial f}{\partial y}(1) \qquad \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}(1) + \frac{\partial f}{\partial y}(-1)$$

$$\frac{\partial f}{\partial s} \cdot \frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right)\left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}\right)$$

$$= \left(\frac{\partial f}{\partial x}\right)^2 - \frac{\partial f}{\partial x}\frac{\partial f}{\partial y} + \frac{\partial f}{\partial x}\frac{\partial f}{\partial y} - \left(\frac{\partial f}{\partial y}\right)^2 \quad \therefore$$

$$\frac{\partial f}{\partial s} \cdot \frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2$$

27)$^{\Delta}$ $\left(2xy + x^2\frac{\partial y}{\partial x}\right) + \left(2y\frac{\partial y}{\partial x}z + y^2\frac{\partial z}{\partial x}\right) + \left(z^2 + 2xz\frac{\partial z}{\partial x}\right) = 0$

$$\frac{\partial y}{\partial x}(x^2 + 2yz) + \frac{\partial z}{\partial x}(y^2 + 2xz) + (2xy + z^2) = 0$$

$$\therefore \boxed{-\frac{2xy + z^2}{y^2 + 2xz}}$$

$\overset{\Delta}{2}$a)

$$e^{xy}\left(\frac{\partial x}{\partial y}y + x\right) + \cos(xz)\left(\frac{\partial z}{\partial y}y + x\right) + 1 = 0$$

$$\left(\frac{\partial x}{\partial y}y + x\right)\left(e^{xy} + \cos(xy)\right) = -1$$

$$\cos(xz)\frac{\partial x}{\partial y}x = -1 - \cos(xz)\frac{\partial x}{\partial y}y - e^{xy}\left(\frac{\partial x}{\partial y}y + x\right)$$

$$\frac{\partial x}{\partial y} = \boxed{-\frac{xe^{xy} + 1}{x\cos(xz)}}$$

31) $(w^2+x^2)^{-1} + (w^2+y^2)^{-1} = 1$

$$-(w^2+x^2)^{-2}\left(2w\frac{\partial w}{\partial y} + \cancel{}\right) - (w^2+y^2)^{-2}\left(2w\frac{\partial w}{\partial y} + 2y\right) = 0$$

$$-2w\frac{\partial w}{\partial y}(w^2+x^2)^{-2} - 2w\frac{\partial w}{\partial y}(w^2+y^2)^{-2} - 2y(w^2+y^2)^{-2} = 0$$

$$-4w\frac{\partial w}{\partial y}\left((w^2+x^2)^{-2} - (w^2+y^2)^{-2}\right) = 2y(w^2+y^2)^{-2}$$

$$\frac{\partial w}{\partial y} = -\frac{\cancel{2}y(w^2+y^2)^{-2}}{2\cancel{4}w\left((w^2+x^2)^{-2} - (w^2+y^2)^{-2}\right)}$$

@ (1, 1, 1), $\left.\frac{\partial w}{\partial y}\right|_{(x,y,w)} = \boxed{-\frac{1}{2}}$

14.7:

1) (a) $f_x(x,y) = 2x - 4y$ $\quad f_y(x,y) = 4y^3 - 4x$

$f_{xx}(x,y) = 2$ $\quad f_{xy} = -4$ $\quad f_{yy}(x,y) = 12y^2$

$2x - 4y = 0$

$4x - 8y = 0$

$4y^3 - 8y = 0$

$4y(y^2 - 2) = 0$

$y = 0,\ y = -2,\ y = 2$

$4y^3 - 4x = 0$

$4y^3 = 4x$

$4(0)^3 = 4x$

$x = 0$

$4(-2)^3 = 4x$

$4x = -32$

$x = -8$

$4(2)^3 = 4x$

$x = 8$

$(0,0), (-2,-8)\ (2,8)$

(0,0)

$D = 2(12(0)^2) - (-4)^2$

$= -16$

saddle point

(-2, -8)

$D = 2(12(-2)^2) - (-4)^2$

$= 48 - 16$

$= 32$ local minimum

(2, 8)

$D = 2(12(2)^2) - (4)^2$

$= 48 - 16 = 32$

local minimum

3) $f_x(x,y) = 2x + y$ $f_y(x,y) = 32y^3 + x - 6y - 3y^2$

$f_{xx}(x,y) = 2$ $f_{yy}(x,y) = 96y^2 - 6 - 6y$ $f_{xy}(x,y) = 1$

$2x + y = 0$ $32y^3 + x - 6y - 3y^2 = 0$

$2x = -y$ $32y^3 - \frac{1}{2}y - 6y - 3y^2 = 0$

$x = -\frac{y}{2}$ $y(32y^2 - \frac{13}{2} - 3y) = 0$

$x = 0, x = \frac{13}{64},$ $y = 0, y = -0.406, y = 0.5$

$x = -\frac{1}{4}$ $-\frac{13}{32}$ $\frac{1}{2}$

$(0,0)$ $(\frac{13}{64}, -\frac{13}{32})$ $(-\frac{1}{4}, \frac{1}{2})$

Saddle point $(0,0)$ | Local Minimum $(\frac{13}{64}, -\frac{13}{32})$ and $(-\frac{1}{4}, \frac{1}{2})$

5) $f_x(x,y) = y^2 - 2yx + y$ $f_y(x,y) = 2xy - x^2 + x$

$f_{xx}(x,y) = -2y$ $f_{xy}(x,y) = 2y - 2x + 1$ $f_{yy}(x,y) = 2x$

$y^2 - 2yx + y = 0$ $2xy - x^2 + x = 0$ $y = \frac{0-1}{2} = -\frac{1}{2}$

$(\frac{x-1}{2})^2 - 2(\frac{x-1}{2})x + (\frac{x-1}{2}) = 0$ $2xy = x^2 - x$

$y = \frac{x-1}{2}$ $y = \frac{-\frac{5}{3} - 1}{2} = -\frac{4}{3}$

$\frac{(x-1)^2}{4} - \frac{2(x-1)}{2}x + \frac{x-1}{2}$ $(0, -\frac{1}{2})$ $(-\frac{5}{3}, -\frac{4}{3})$

$\frac{1}{4}(x-1)^2 - (x^2 - x) + \frac{1}{2}(x-1) = 0$

$\frac{1}{4}(x^2 - 2x + 1) - 4x^2 - 4x + \frac{1}{4}(x-1) = 0$

$-3x^2 - 5x = 0$

$x(-3x - 5) = 0$

$x = 0, x = -\frac{5}{3}$

a) $-\frac{1}{2}(-\frac{1}{2} - 2(0) + 1) = 0$

0 (using $2y - x + 1$) $= 0$

b) critical at $(0, -\frac{1}{2})$, $(-\frac{5}{3}, -\frac{4}{3})$

c) $(0, -\frac{1}{2})$ saddle

$(-\frac{5}{3}, -\frac{4}{3})$ saddle.

7) $f_x(x,y) = 2x - y + 1$ $\quad f_y(x,y) = 2y - x$

$f_{xx}(x,y) = 2$ $\quad f_{yy}(x,y) = 1$ $\quad f_{xy} = -1$

$2x - y + 1 = 0$ $\quad 2y - x = 0$ $\quad D = 2 \cdot 1 - (-1)^2 = 1$

$4y - y + 1 = 0 \leftarrow 2y = x$

$3y = -1$ $\quad 2(-\frac{1}{3}) = x$

$y = -\frac{1}{3}$ $\quad x = -\frac{2}{3}$

$(-\frac{2}{3}, -\frac{1}{3}) \rightarrow$ local minimum

11) $f_x(x,y) = 4 - 9x^2 - 2y^2$ $\quad f_y(x,y) = -4xy$

$f_{xx}(x,y) = -18x$ $\quad f_{yy}(x,y) = -4x$ $\quad f_{xy}(x,y) = -4y$

$4 - 9x^2 - 2y^2 = 0$ $\quad -4xy = 0$

$x = \frac{0}{-4y} = 0$ $\quad D = 0(4\sqrt{2}) - (4\sqrt{2})^2 = -32$

$4 - 2y^2 = 0$

$y = -1.41, y = 1.41$ $\quad -4x\sqrt{2} = 0$

$-\sqrt{2}$ $\quad \sqrt{2}$

$(0, -\sqrt{2})$ $(0, \sqrt{2}) \Rightarrow$ saddle point

$4 - 9x^2 = 0$ $\quad -4(\frac{2}{3})y = 0$ $\quad D = (-18(\frac{2}{3}))(-4(\frac{2}{3})) - 0$

$-9x^2 = -4$ $\quad y = 0$ $\quad D = 32$

$x = \sqrt{\frac{4}{9}} = \pm\frac{2}{3}$

$(\frac{2}{3}, 0) \rightarrow$ local maximum

$(-\frac{2}{3}, 0) \rightarrow$ local minimum

13) $f_x(x,y) = 4x^3 - 4y$ $f_y(x,y) = 4y^3 - 4x$

$f_{xx}(x,y) = 12x^2$ $f_{yy}(x,y) = 12y^2$ $f_{xy} = -4$

$4x^3 - 4y = 0$ $4y^3 - 4x = 0$ $(0,0)$ $(1,1)$ $(-1,-1)$

$(0,0)$: $D = 12(0)^2 12(0)^2 - (-4)^2 = -16$ **$(0,0) \to$ Saddle Point**

$(1,1)$: $D = 12(1)^2 12(1)^2 - (-4)^2 = 128$ **$(1,1) \to$ Local Minimum**

$f_{xx}(1,1) = 12(1)^2 = 12$

$(-1,-1)$: $D = 12(-1)^2 12(-1)^2 - (-4)^2 = 128$ **$(-1,-1) \to$ Local Minimum**

$f_{xx}(-1,-1) = 12(-1)^2 = 12$

19) $f_x(x,y) = x^{-1} - 1$ $f_y(x,y) = 2y^{-1} - 4$

$f_{xx}(x,y) = -x^{-2}$ $f_{yy}(x,y) = -2y^{-2}$ $f_{xy}(x,y) = 0$

$x^{-1} - 1 = 0$ $2y^{-1} - 4 = 0$

$x = 1$ $y = \frac{1}{2}$ $(1, \frac{1}{2})$

$(1, \frac{1}{2})$: $D = (-(1)^{-2})(-2(\frac{1}{2})^{-2}) - 0^2 = 8$

$f_{xx}(1, \frac{1}{2}) = -(1)^{-2} = -1$ **$(1, \frac{1}{2}) \to$ Local Maximum**

21) $f_x(x,y) = 1 - (x+y)^{-1}$ $f_y = -2y - (x+y)^{-1}$

$f_{xx}(x,y) = (x+y)^{-2}$ $f_{yy}(x,y) = -2 + (x+y)^{-2}$ $f_{xy} = (x+y)^{-2}$

$1 - (x+y)^{-1} = 0$ $-2y - (1 - y + y)^{-1} = 0$

$1 = (x+y)^{-1}$ $-2y - (1)^{-1} = 0$

$(x+y) = 1$ $-2y = 1$ $D = (\frac{3}{2} - \frac{1}{2})^{-2}(-2 + (\frac{3}{2} - \frac{1}{2})^{-2})$

$x = 1 - y$ $y = -\frac{1}{2}$ $-((\frac{3}{2} - \frac{1}{2})^{-2})^2$

$x = 1 - (-\frac{1}{2}) = \frac{3}{2}$ $= -2$

$(\frac{3}{2}, -\frac{1}{2}) \longrightarrow$ Saddle Point

23) $f_x(x,y) = e^{y-x^2} + (x+3y)e^{y-x^2}(-2x)$

$f_y(x,y) = 3e^{y-x^2} + (x+3y)e^{y-x^2}$

$f_{xx}(x,y) =$ ~~$-2x\,e^{y-x^2} \ldots$~~

$4e^{y-x^2}x^3 + 12e^{y-x^2}yx^2 - 6e^{y-x^2}x - 6e^{y-x^2}y$

I dont know how to solve this problem.

29) $f(0,0) = 0$ $f(1,0) = 1$

$f(0,1) = 1$ $f(1,1) = 2$

Maximum Value 2. at (1, 1)
Minimum Value 0 at (0, 0)

35) $f(0,0) = 0$

$f(0,2) = 0 + 2 - 0 - 4 - 0 = -2$

$f(2,0) = 2 + 0 - 4 - 0 - 0 = -2$

$f(x,y)$ smaller when x an y larger

$x + y - x^2 - y^2 - xy$

$-(x^2 + xy + y^2) + (x + y)$

Max at (0, 0) ?