HW due 10/11/20

14.6 : 1, 3, 5, 7, 15, 17*, 23, 27, 29, 31

14.7 : 1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35

14.6

1a. $\frac{df}{dx} = 2xy^3 \quad \frac{df}{dy} = 3x^2y^2 \quad \frac{df}{dz} = 4z^3$

1b. $\frac{dx}{ds} = 2s \quad \frac{dy}{ds} = t^2 \quad \frac{dz}{ds} = 2st$

1c. $\frac{df}{ds} = (2xy^3)(2s) + (3x^2y^2)(t^2) + (4z^3)(2st)$

$= 4xy^3s + 3x^2y^2t^2 + 8z^3st$

$= 4(s^2)(st^2)^3(s) + 3(s^2)^2(st^2)^2(t^2) + 8(s^2t)^3(st)$

$= 4s^6t^6 + 3s^6t^6 + 8s^7t^4$

$= \boxed{7s^6t^6 + 8s^7t^4}$

3. $f(x,y,z) = xy + z^2 \quad x = s^2 \quad y = 2rs \quad z = r^2$

$\frac{df}{dx} = y \quad \frac{df}{dy} = x \quad \frac{df}{dz} = 2z$

$\frac{dx}{ds} = 2s \quad \frac{dy}{ds} = 2r \quad \frac{dz}{ds} = 0$

$\frac{df}{ds} = (y)(2s) + (x)(2r) + (2z)(0) = (2rs)(2s) + (s^2)(2r) + 0$

$= 4rs^2 + 2s^2r = \boxed{6s^2r}$

$\frac{dx}{dr} = 0 \quad \frac{dy}{dr} = 2s \quad \frac{dz}{dr} = 2r$

$\frac{df}{dr} = (y)(0) + (x)(2s) + (2z)(2r) = 0 + (s^2)(2s) + (2r^2)(2r)$

$= \boxed{2s^3 + 4r^3}$

5. $g(x,y) = \cos(x-y) \quad x = 3u - 5v \quad y = -7u + 15v$

$\frac{dg}{dx} = -\sin(x-y) \quad \frac{dg}{dy} = \sin(x-y)$

$\frac{dx}{du} = 3 \quad \frac{dy}{du} = -7$

$\frac{dg}{du} = (-\sin(x-y))(3) + (\sin(x-y))(-7) = \boxed{-10\sin(10u-20v)}$

$\frac{dx}{dv} = -5 \quad \frac{dy}{dv} = 15$

$\frac{dg}{dv} = (-\sin(x-y))(-5) + (\sin(x-y))(15) = \boxed{20\sin(10u-20v)}$

7. $F(u,v) = e^{u+v} \quad u = x^2 \quad v = xy$

$\frac{dF}{du} = e^{u+v} \quad \frac{dF}{dv} = e^{u+v}$

$\frac{du}{dy} = 0 \quad \frac{dv}{dy} = x$

$\frac{dF}{dy} = xe^{u+v} = \boxed{xe^{x^2+xy}}$

15. $g(x,y) = x^2 - y^2 \quad x = e^u\cos v \quad y = e^u \sin v$

$\frac{dg}{dx} = 2x \quad \frac{dg}{dy} = -2y$

$\frac{dx}{du} = e^u\cos v \quad \frac{dy}{du} = e^u \sin v$

$\frac{dg}{du} = (2x)(e^u\cos v) + (-2y)(e^u\sin v)$

$= (2e^u\cos v)(e^u\cos v) + (-2e^u\sin v)(e^u\sin v)$

$= 2e^{2u}\cos^2 v - 2e^{2u}\sin^2 v$

$= 2e^{2u}(\cos^2 v - \sin^2 v) = 2e^{2u}\cos 2v$

$= \boxed{2\cos(2)}$

23. $x = s+t \quad y = s-t$

$\frac{dx}{ds} = 1 \quad \frac{dy}{ds} = 1 \quad \frac{dx}{dt} = 1 \quad \frac{dy}{dt} = -1$

$\frac{df}{ds} = \frac{df}{dx}\cdot 1 + \frac{df}{dy}\cdot 1 = \frac{df}{dx} + \frac{df}{dy}$

$\frac{df}{dt} = \frac{df}{dx}\cdot\frac{dx}{dt} + \frac{df}{dy}\cdot\frac{dy}{dt}$

$\frac{df}{dt} = \frac{df}{dx}\cdot 1 + \frac{df}{dy}(-1) = \frac{df}{dx} - \frac{df}{dy}$

$\frac{df}{ds}\cdot\frac{df}{dt} = \left(\frac{df}{dx} + \frac{df}{dy}\right)\left(\frac{df}{dx} - \frac{df}{dy}\right) = \left(\frac{df}{dx}\right)^2 - \left(\frac{df}{dy}\right)^2$

27. $f(x,y,z) = x^2y + y^2z + z^2x - 10$

$f_x = 2xy + z^2 \quad f_y = x^2 + 2yz \quad f_z = y^2 + 2zx$

$\frac{dz}{dx} = \frac{-f_x}{f_z} = \frac{-(2xy + z^2)}{y^2 + 2zx} = \boxed{\frac{-2xy - z^2}{y^2 + 2xz}}$

29. $f(x,y,z) = e^{xy} + \sin(xz) + y$

$f_x = ye^{xy} + z\cos(xz) \quad f_y = xe^{xy} + 1 \quad f_z = x\cos(xz)$

$\frac{dz}{dy} = \frac{-f_y}{f_z} = \boxed{\frac{-(xe^{xy}+1)}{x\cos(xz)}}$

31. $F(x,y,w) = \frac{1}{w^2+x^2} + \frac{1}{w^2+y^2} - 1$

$F_y = 0 + \frac{(w^2+y^2)\cdot 0 - 1\cdot 2y}{(w^2+y^2)^2} = \frac{-2y}{(w^2+y^2)^2}$

$F_w = \frac{-2w}{(w^2+y^2)^2} - \frac{2w}{(w^2+x^2)^2}$

$\frac{dw}{dy} = -\frac{y(w^2+x^2)^2}{w((w^2+y^2)^2 + (w^2+x^2)^2)} \quad$ at $(1,1,1)$

$= \boxed{-\frac{1}{2}}$

14.7 : 1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35

14.7

3. $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_x = 2x + y \quad f_y = 32y^3 + x - 6y - 3y^2 \quad x = -\frac{y}{2}$

$32y^3 - \frac{y}{2} - 6y - 3y^2 = 0$

$y = 0, \frac{-13}{32}, \frac{1}{2} \quad x = 0, \frac{13}{64}, \frac{-1}{4}$

critical points: $(0,0)\ (\frac{13}{64}, \frac{-13}{32})\ (-\frac{1}{4}, \frac{1}{2})$

$(0,0)$: saddle point $(\frac{13}{64}, \frac{-13}{32})$: local min

$(\frac{-1}{4}, \frac{1}{2})$ local max

5. $f(x,y) = y^2x - yx^2 + xy$

$f_x = y^2 - 2yx + y \quad f_y = 2yx - x^2 + x$

critical points: $(0,0)\ (0,-1)\ (\frac{1}{2}, -\frac{1}{3})\ (1,0)$

saddle points: $(0,0)\ (1,0)\ (0,-1)$

local min: $(\frac{1}{3}, -\frac{1}{3})$

7. $f(x,y) = x^2 + y^2 - xy + x$

$f_x = 2x - y + 1 \quad f_y = 2y - x$

critical point $(-\frac{2}{3}, -\frac{1}{3})$ ← local min

11. $f(x,y) = 4x - 3x^3 - 2xy^2$

$f_x = 4 - 9x^2 - 2y^2 \quad f_y = -4xy$

critical points: $(0,\sqrt{2})(0,-\sqrt{2})(\frac{2}{3},0)(-\frac{2}{3},0)$

saddle points: $(0,\sqrt{2})(0,-\sqrt{2})$

local max: $(\frac{2}{3},0)$ local min: $(-\frac{2}{3},0)$

13. $f(x,y) = x^4 + y^4 - 4xy$

$f_x = 4x^3 - 4y \quad f_y = 4y^3 - 4x$

critical points $(0,0)\ (1,1)\ (-1,-1)$

saddle point $(0,0)$

local min: $(1,1)\ (-1,-1)$

19. $f(x,y) = \ln x + 2\ln y - x - 4y$

$f_x = \frac{1}{x} - 1 \quad f_y = \frac{2}{y} - 4$

critical point: $(1, \frac{1}{2})$ ← local max

21. $f(x,y) = x - y^2 - \ln(x+y)$

$f_x = 1 - \frac{1}{x+y} \quad f_y = -2y - \frac{1}{x+y}$

critical point: $(\frac{3}{2}, -\frac{1}{2})$ ← saddle point

23. $f(x,y) = (x+3y)e^{y-x^2}$

$f_x = (1 - 6xy - 2x^2)e^{y-x^2}$

$f_y = (x + 3y + 3)e^{y-x^2}$

critical point: $(-\frac{1}{6}, -\frac{17}{18})$ ← local min

29. $f(x,y) = x + y$

$f_x = 1 \quad f_y = 1$

$f(0,0) = 0 \quad f(1,0) = 0$

$f(1,0) = 1 \quad f(1,1) = 2$

$f(0,1) = 1$

abs max: $(1,1)$ abs min: $(0,0)\ (1,0)$

35. $f(x,y) = x + y - x^2 - y^2 - xy$

$f_x = 1 - 2x - y \quad f_y = 1 - 2y - x$

a. critical point: $(\frac{1}{3}, \frac{1}{3})$

$f(\frac{1}{3}, \frac{1}{3}) = \frac{1}{3}$

b. critical point $(\frac{1}{2}, 0)$

$f(\frac{1}{2}, 0) = \frac{1}{4}$

c. critical point $(0, \frac{1}{2})\ (2, -\frac{1}{2})\ (-\frac{1}{2}, 2)$

$f(0, \frac{1}{2}) = \frac{1}{4} \quad f(2, -\frac{1}{2}) = -\frac{7}{4} \quad f(-\frac{1}{2}, 2) = -\frac{7}{4}$

d. $f(\frac{1}{3}, \frac{1}{3}) = \frac{1}{3}$ ← largest value