Sai Embar

14.6-14.7 (Oct. 11th)

14.6: # 1, 3, 5, 7, 15, 23, 27, 29, 31

14.7: # 1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35

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1) a) $\frac{\partial}{\partial x}(x^2y^3) = \underline{2xy^3}$, $\frac{\partial}{\partial y}(x^2y^3) = \underline{3x^2y^2}$, $\frac{\partial}{\partial z}(x^2y^3) = \underline{4z^3}$

b) $\frac{\partial x}{\partial s} = 2s$, $\frac{\partial y}{\partial s} = t^2$, $\frac{\partial z}{\partial s} = 2st$

c) $\frac{\partial f}{\partial s} = 2xy^3(2s) + 3x^2y^2(t^2) + 4z^3(2st)$

$= \boxed{7s^6t^6 + 8s^7t^4}$

3) $\frac{\partial f}{\partial s}(s^2(2rs)) = \frac{\partial f}{\partial s}(2rs^3) = \boxed{\frac{\partial f}{\partial s} = 6rs^2}$

$\frac{\partial f}{\partial r}(s^2(2rs)) + r^2) = \frac{\partial f}{\partial r}(2rs^3 + r^2) = \boxed{\frac{\partial f}{\partial r} = 2s^3 + 4r^3}$

5) $\frac{\partial g}{\partial u}(\cos(3u-5v) - (-7u+5v)) = \boxed{\frac{\partial g}{\partial u} = -10\sin(10u-20v)}$

$\frac{\partial g}{\partial v}(\cos(3u-5v) - (-7u+5v)) = \boxed{\frac{\partial g}{\partial v} : 20\sin(10u-20v)}$

7) $\frac{\partial f}{\partial y}(e^{x^2+xy}) = \boxed{xe^{x^2+xy}}$

15) $\frac{\partial g}{\partial u}$ at $(u, v) = (0, 1)$

$g(u,v) = (e^u\cos v)^2 - (e^u\sin v)^2$

$\frac{\partial}{\partial u} = \cos(2v)e^{2u} \cdot 2$

$= \boxed{2\cos(2)}$

23) $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\left(\frac{\partial x}{\partial s}\right) + \frac{\partial f}{\partial y}\left(\frac{\partial y}{\partial s}\right)$

$\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\left(\frac{\partial x}{\partial t}\right) + \frac{\partial f}{\partial y}\left(\frac{\partial y}{\partial t}\right)$

$\underbrace{\frac{\partial x}{\partial s}, \frac{\partial x}{\partial t}, \frac{\partial y}{\partial s}, \frac{\partial y}{\partial t}}_{=1} = 1$

$\left.\begin{aligned} \frac{\partial f}{\partial s} &= \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial t} &= \frac{\partial f}{\partial x} - \frac{\partial f}{\partial y} \end{aligned}\right\}$ multiply

$\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}\right) = \boxed{\left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2}$

27) $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$

$F_x = 2xy + z^2, \quad F_z = y^2 + 2xz$

$$\frac{\partial z}{\partial x} = -\frac{2xy + z^2}{2xz + y^2}$$

29) $\partial z = \frac{F_y}{F_z}$

$F_y = xe^{xy} + 1, \quad F_z = x\cos(xz)$

$$\frac{\partial z}{\partial y} = -\frac{xe^{xy}+1}{x\cos(xz)}$$

31) $\frac{\partial w}{\partial y} = -\frac{F_y}{F_w}$

$F_y = \frac{-2y}{(w^2+y^2)^2}, \quad f_w = \frac{-2w}{(w^2+y^2)^2} \quad \frac{-2w}{(w^2+y^2)^2}$

$\frac{\partial w}{\partial y} = \frac{-y(w^2+x^2)^2}{w((w^2+y^2)^2+(w^2+x^2)^2)}$ at $(1,1,1) \to \frac{\partial w}{\partial y} = -\frac{1}{2}$

14.7: # 1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35

1) $f_x(x,y) = 2x - 4y$

$f_x(x,y) = 0$

$f_x(a,b) = 0$

$2a - 4b = 0, \quad a = 2b$

$f_y(x,y) = 0, \quad f_y(x,y) = 4y^3 - 4x$

$f_y(a,b) = 0, \quad 4b^3 - 4a = 0$

$b^3 = a \cancel{x}^2$

$b^3 = 2b, \quad b(b^2 - 2) = 0$

$(0,0), \ (2\sqrt{2}, \sqrt{2}), \ (-2\sqrt{2}, -\sqrt{2})$

b) The abs min is -4

$f_{xx} = 2, \quad f_{yy} = 12y^2$

$f_{xy} = -4$

3) $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_y(x,y) = 32y^3 + x - 6y - 3y^2 = 0$

$f_x(x,y) = 2x + y = 0$

$y = -2x$

$f_y(x,y) = 32(-2x)^3 + x - 6(-2x) - 3(2x)^2 = 0$

$256x^3 + x + 12x - 12x^2 = 0$

$x(256x^2 + 12x - 13) = 0$

$x = \frac{-12 \pm \sqrt{144 - 4(-13)(256)}}{512}$

$x = \frac{-12 \pm 116}{512} \to x = \frac{13}{64}$ or $-\frac{1}{4}$

$x: 0, -\frac{1}{4}, \frac{13}{64}$

$(0,0), \left(\frac{13}{64}, \frac{-13}{32}\right), \left(-\frac{1}{4}, \frac{1}{2}\right)$

5) a) $f(x,y) = y^2x - yx^2 + xy$

$f_x = 0,\ f_y = 0$

$f_x = y^2 - 2xy + y = 0 \rightarrow y(y - 2x + 1) = 0$

$f_y = 2yx - x^2 + x = 0 \rightarrow x(2y - x + 1) = 0$

b) $y = 0$ or $y - 2x + 1 = 0 \rightarrow y = 2x - 1$

$x(-x+1) = 0 \rightarrow x = 0$ or 1

$x(4x - 2 - x + 1) = 0$

$x(3x - 1) = 0 \rightarrow x = 0$ or $x = \frac{1}{3}$

$(0,-1),\ (\frac{1}{3}, -\frac{1}{3})$

c) $f_{xx} = -2y$ $\qquad f_{xx} f_{yy} - f_{xy}^2 = (-2y)(2x) - (2y - 2x + 1)^2$

$f_{yy} = 2x$ $\qquad = -4xy - (2y - 2x + 1)^2$

$f_{xy} = 2y - 2x + 1$

$D(0,0) = -1 < 0$

$D(1,0) = -1 < 0$

$D(0,-1) = -1 < 0$

$D(\frac{1}{3}, -\frac{1}{3}) = -4(\frac{1}{3})(-\frac{1}{3}) - (-\frac{2}{3} - \frac{2}{3} + 1)^2 = \frac{1}{3} > 0$

$f_{xx}(\frac{1}{3}, -\frac{1}{3}) = \frac{2}{3} > 0$

at $(0,0)$, $(1,0)$, $(0,-1)$ are saddle points and $(\frac{1}{3}, -\frac{1}{3})$ local min

7) $f_x(x,y) = 2x - y + 1 = 0$

$= 2y - x = 0$

$2(2y) - y + 1 = 0$; $3y + 1 = 0$

$y = -\frac{1}{3}$

$x = -\frac{2}{3}$, critical point is $(-\frac{2}{3}, -\frac{1}{3})$

$f_{xx}(x,y) = 2$ $\qquad D(x,y) = -1$

$f_{yy}(x,y) = 2$ $\qquad D(x,y) = 2 \cdot 2 - (-1)^2 = 3$

$f_{xy}(x,y) = -1$ $\qquad D(-\frac{2}{3}, -\frac{1}{3}) = 3 > 0$ and $f_{xx}(-\frac{2}{3}, -\frac{1}{3}) > 0$ local min

11) $f_x = 4 - 9x^2 - 2y^2$

$f_y = -4xy$

$f_y = 0$

$f_x = 0$ gives $y = \pm\sqrt{2}$

crit points $(0, \sqrt{2})$, $(0, -\sqrt{2})$, $(\frac{2}{3}, 0)$ and $(-\frac{2}{3}, 0)$

saddle point because $D < 0$.

local max $D > 0$, $f_{xx} < 0$

local min $D > 0$, $f_{xx} > 0$

$f_{xx} = -18x$

$f_{yy} = -4x$

$f_{xy} = -4y$

13) $f_x = 4x^3 - 4y = 0$, $y = x^3$

$f_y = 4y^3 - 4x = 0$

$4(x^3)^3 - 4x = 0$

$4x(x^8 - 1) = 0$

$x = 0$ and $x = \pm 1$

$x = 0 \therefore y = 0$ and $(0,0)$

$x = \pm 1 \therefore y = \pm 1$ at $(1,1)$

$x = -1 \therefore y = -1$ at $(-1,-1)$

$f_{xx} = 12x^2$, $f_{yy} = 12y^2$, $f_{xy} = -4$

$D(x,y) = 144x^2y^2 - 16$

$D(1,1) = 128 > 0$, $f_{xx} > 0$

$D(-1,-1) = 128 > 0$, $f_{xx} > 0$

$D(0,0) = -16 < 0$, $f_{xx} > 0$

$f(1,1)$, $f(-1,-1)$ is a local min

so $(0,0)$ is ~~a saddle point~~

19) $f_x = \frac{1}{x} - 1 = -1$

$f_y = -\frac{2}{y} - 4 = -8$

$f_x = 0$, $f_y = 0$

$f_{xx} = -\frac{1}{x^2}$

$f_{yy} = -\frac{2}{y^2}$

$f_{xy} = 0$

crit point: $(1, \frac{1}{2})$

$D > 0$, $f_{xx} < 0$

21) $f_x = 1 - \frac{1}{x+y}$

$f_y = -2y + \frac{1}{x+y}$

$f_x = 0$, $f_y = 0$

$(\frac{3}{2}, -\frac{1}{2})$

$f_{xx} = \frac{1}{(x+y)^2}$

$f_{yy} = -2 + \frac{1}{(x+y)^2}$

$f_{xy} = \frac{1}{(x+y)^2}$

crit point: $(\frac{3}{2}, -\frac{1}{2})$

23) $f(x,y) = (x+3y)e^{y-x^2}$

$f_x(x,y) = (1-2x^2-6xy)e^{y-x^2} = 0$

$f_y(x,y) = (3+x+3y)e^{y-x^2} = 0$

$e^{y-x^2} \neq 0$, $(1-2x^2-6xy) = 0$

$(3+x+3y) = 0$

crit point is $\left(-\frac{1}{6}, -\frac{17}{18}\right)$

$f_{xx} = (2x^3+6x^2y-3x-3y)\,2e^{y+x^2}$

$f_{yy} = (6+x+3y)e^{y-x^2}$

$f_{xy} = (1-6xy-2x^2-6x)\,e^{y-x^2}$

29) max when x=1 and y=1

min when x=0 and y=0

$f(1,1) = 1+1 = 2$

$f(0,0) = 0+0 = 0$

global max = 0

global min = 2

35) $f_x = 1-2x-y = 0 \rightarrow -2x-2y = -1 \rightarrow 2x+y = 1$

$f_y = 1-2y-x = 0 \rightarrow -x-2y = -1 \rightarrow x+2y = 1$

$x = 1/3,\ y = 1/3$

$f_{xx} = -2,\ f_{yy} = -2,\ f_{xy} = -1$

$f_{xx}\left(\frac{1}{3}, \frac{1}{3}\right) = -2 < 0$

$D = f_{xx} \cdot f_{yy} - (f_{xy})^2$

$= (-2)(-2) - (-1)^2$

$= 4-1,\ 3$, so $D > 0$

max at $\left(\frac{1}{3}, \frac{1}{3}\right)$ and its value is; $\frac{1}{3} + \frac{1}{3} - \left(\frac{1}{3}\right)^2 - \frac{1}{9} = \boxed{\frac{1}{3}}$