HW Due 10/11

14.6

Rahul Pakeja

14.6 - #1, 3, 5, 7, 15, 17 (optional), 23, 27, 29, 31:

(1) a) $\frac{df}{dx} = 2y^3x$ $\frac{df}{dy} = 3x^2y^2$ $\frac{df}{dz} = 4z^3$

b) $\frac{dx}{ds} = 2s$ $\frac{dy}{ds} = t^2$ $\frac{dz}{ds} = 2ts$

$$\frac{df}{ds} = 2y^3x \cdot 2s + 3x^2y^2(t^2) + 4z^3(2ts)$$

$$= 4y^3xs + 3x^2y^2t^2 + 8tsz^3$$

Plug In values of x, y, z

$$= 4(st^2)^3(s^2)s + 3(s^2)^2(st^2)^2t^2 + 8ts(s^2t)^3$$

$$= 4s^3t^6s^3 + 3s^4s^2t^4t^2 + 8ts(s^6)(t^3)$$

$$= 4 \cdot t^6s^6 + 3s^6t^6 + 8t^4s^7$$

$$= \boxed{7s^6t^6 + 8s^7t^4}$$

(3) $\frac{df}{dx} = y$ $\frac{df}{dy} = x$ $\frac{df}{dz} = 2z$

$\frac{dx}{ds} = 2s$ $\frac{dy}{ds} = 2r$ $\frac{dz}{ds} = 0$ $\frac{dx}{dr} = 0$ $\frac{dy}{dr} = 2s$ $\frac{dz}{dr} = 2r$

$$\frac{df}{ds} = y(2s) + x(2r) + (2z)(0)$$

$$= 2ys + 2rx = 2(2rs)s + 2r(s^2)$$

$$= 4rs^2 + 2rs^2 = \boxed{6rs^2}$$

$$\frac{df}{dr} = y(0) + x(2s) + 2z(2r)$$

$$= 0 + 2xs + 4zr = 2(s^2) + 4(r^2)(r)$$

$$= \boxed{2s^2 + 4r^3}$$

(5) $\frac{dg}{dx} = -\sin(x-y)$ $\frac{dg}{dy} = \sin(x-y)$

$\frac{dx}{du} = 3$ $\frac{dy}{du} = -7$

$\frac{dx}{dv} = -5$ $\frac{dy}{dv} = 15$

$\frac{dg}{du} = -3\sin(x-y) - 7\sin(x-y) = -10\sin(x-y) = \boxed{-10\sin(10u-20v)}$

$\frac{dg}{dv} = 5\sin(x-y) + 15\sin(x-y) = 20\sin(x-y) = \boxed{20\sin(10u-20v)}$

(7) $\frac{df}{du} = e^{u+v}$ $\frac{df}{dv} = e^{u+v}$

$\frac{du}{dy} = 0$ $\frac{dv}{dy} = x$

$\frac{df}{dy} = xe^{u+v} = \boxed{xe^{x^2+xy}}$

(15) $\frac{dg}{dx} = 2x$ $\frac{dg}{dy} = -2y$

$\frac{dx}{du} = \cos v(e^u)$ $\frac{dy}{du} = (\sin v)e^u$

$\frac{dg}{du} = 2x\cos(v)e^u - 2y\sin(v)e^u$

$= 2(e^u\cos(v))\cos(v)e^u - 2(e^u\sin v)\sin(v)e^u$

$= 2(e^0\cos(1))\cos(1)e^0 - 2(e^0\sin(1))\sin(1)e^0$

$= 2\cos^2(1) - 2\sin^2(1) = 2(\cos^2(1) - \sin^2(1))$

$= \boxed{2\cos(2)}$ identity

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14.6 - #23, 27, 29, 31:

(23) $F_x = z^2 + y$ $\quad F_y = 2zy + x$ $\quad F_z = 2xz + y^2$

$$\frac{dz}{dx} = -\frac{F_x}{F_z} = \frac{-(z^2+y)}{(2xz+y^2)} \qquad \frac{dz}{dy} = \frac{-F_y}{F_z} = \frac{-(2zy+x)}{(2xz+y^2)}$$

(27) $\frac{dz}{dx} = 2yx + y^2(z') + x(2zz') + z^2 = 0$

$u = y^2 \quad v = z \qquad u = x \quad v = z^2$

$u' = 0 \quad v' = z' \qquad v = 1 \quad v' = 2zz'$

$$\frac{z'(2xz+y^2)}{2xz+y^2} = -z^2 - 2xy$$

$$\boxed{z' = \frac{-z^2 - 2xy}{y^2 + 2xz}}$$

(29) $xe^{xy} + \cos(xz) \cdot xz' + 1$

$u = x \quad v = z$

$u' = 0 \quad v' = z'$

$$\frac{z'(x\cos(xz))}{x\cos(xz)} = -xe^{xy} - 1$$

$$\boxed{z' = \frac{-xe^{xy} - 1}{x\cos(xz)}}$$

(31) $\frac{-1}{(w^2+x^2)^2} \cdot (2ww') - \frac{1}{(w^2+y^2)^2} \cdot (2ww' + 2y) = 0$

$$\frac{-1 \cdot (2w')}{(4)} - \frac{1}{4} \cdot (2w' + 2) = 0$$

$$4\left(-\frac{w'}{2} - \frac{(2w'+2)}{4} = 0\right)$$

$$-2w' - (2w' + 2) = 0$$

$$-2w' - 2w' - 2 = 0$$

$$\frac{-4w'}{-4} = \frac{2}{-4} \qquad \boxed{w' = -\tfrac{1}{2}}$$

HW Due 10/11
14.7

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14.7 - #1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35:

① a) $f_x = 2x - 4y = 0$ $\quad f_x(a,b) = 2a - 4b = 0$

$\frac{2a}{2} = \frac{4b}{2}$ $\quad \boxed{a = 2b}$

$f_y = 4y^3 - 4x = 0$ $\quad f_y(a,b) = 4b^3 - 4a = 0$

$\frac{4b^3}{4} = \frac{4a}{4}$

$b^3 = a$ $\quad b^3 = 2b$
$\quad -2b$

$b^3 - 2b = 0$ $\quad b(b^2 - 2) = 0$

$b = 0$ or $\pm\sqrt{2}$

Thus points are: $(0,0)$, $(2\sqrt{2}, \sqrt{2})$, $(-2\sqrt{2}, -\sqrt{2})$

b) $f_{xx} = 2$ $\quad f_{yy} = 12y^2$
$f_{xy} = -4$

$D = 2(12y^2) - (-4)^2 = 24y^2 - 16$

$(0,0) = -16 < 0 \rightarrow (0,0)$ is a Saddle Point

$(2\sqrt{2}, \sqrt{2}) = 24(2) - 16 = 8$ & $f_{xx} = 0 > 2 \rightarrow (2\sqrt{2}, \sqrt{2}) \rightarrow$ local min.

$(-2\sqrt{2}, -\sqrt{2}) = 8$ & $f_{xx} > 0 \rightarrow (-2\sqrt{2}, -\sqrt{2}) \rightarrow$ local min

③ $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_x = 2x + y$ $\quad f_y = 32y^3 + x - 6y - 3y^2$

$f_{xx} = 2$

$f_{xy} = 1$ $\quad f_{yy} = 96y^2 - 6 - 6y$

$2x + y = 0$ $\quad 32y^3 + x - 6y - 3y^2 = 0$

$y = -2x$ $\quad 32(-2x)^3 + x - 6(-2x) - 3(-2x)^2 = 0$

$-256x^3 + x + 12x - 12x^2 = 0$

Critical Points:

$x(-256x^2 - 12x + 13) = 0$

$x = 0$ & $x = \frac{-12 \pm \sqrt{144 - 4(-13)(256)}}{512}$

$(0,0)$

$(\frac{13}{64}, -\frac{13}{32})$ $\quad x = \frac{-12 \pm 116}{512} \rightarrow x = \frac{13}{64}, \frac{-1}{4}$

$(-\frac{1}{4}, \frac{1}{2})$

(0,0) reflects a saddle point as it is neither increasing or decreasing
($\frac{13}{64}$, $\frac{-13}{32}$) and ($\frac{-1}{4}$, $\frac{1}{2}$) are local mins.

⑤ $f(x,y) = y^2x - yx^2 + xy$

a) $f_x = y^2 - 2yx + y = 0 \to y(y - 2x + 1) = 0$

$f_y = 2xy - x^2 + x = 0 \to x(2y - x + 1) = 0$

b) $f_x = 0$ $\quad y = 0$ or $y = 2x - 1$

Substitute into f_y: $\quad y = 0 \quad x(-x+1) = 0$

$x = 0$ or $x = 1$

Critical Points:

$(0,0)$ or $(1,0)$ $\quad (0,-1)$ $\quad (\frac{1}{3}, -\frac{1}{3})$

$2x+1$ into 2nd equation

$x(4x - 2 - x + 1) = 0$

$x(3x - 1) = 0 \qquad x = 0$ or $x = \frac{1}{3}$

$f_{xx} = -2y$

$f_{yy} = 2x$

$f_{xy} = 2y - 2x + 1$

$D = (-2y)(2x) - (2y - 2x + 1)^2$

$= -4xy - (2y - 2x + 1)^2$

$D(0,0) = -1 < 0 \to (0,0) \to$ Saddle point

$D(1,0) = -1 < 0 \to (1,0)$ is a saddle point

$D(0,-1) = -1 < 0 \to (0,-1) \to$ Saddle point

$D(\frac{1}{3}, -\frac{1}{3}) = \frac{1}{3} > 0$ & $f_{xx}(\frac{1}{3}, -\frac{1}{3}) \to \frac{2}{3} > 0$

Thus $(\frac{1}{3}, -\frac{1}{3})$ is a local minimum

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14.7

Rahul Paleja

14.7 – #7, 11, 13, 19, 21, 23, 29, 35:

(7) $f(x,y) = x^2 + y^2 - xy + x$

$f_x = 2x - y + 1 = 0$ $\quad 3y = -1 \quad y = -1/3$

$f_y = 2y - x = 0 \quad x = 2y$

Critical Point:
$(-2/3, -1/3)$

$f_{xx} = 2$
$f_{yy} = 2$
$f_{xy} = -1$

$D = 2(2) - (-1)^2 = 3 > 0$ at $(-2/3, -1/3)$
$+ f_{xx} = 2 > 0$, thus $(-2/3, -1/3)$ is a local minimum

(11) $f(x,y) = 4x - 3x^3 - 2xy^2$

$f_x = 4 - 9x^2 - 2y^2 = 0$
$f_y = -4xy = 0$

when $x=0$: $(0, \sqrt{2}), (0, -\sqrt{2})$ $\quad$ when $y=0$: $(2/3, 0)\ (-2/3, 0)$

$x = 0$ or $y = 0$

$f_{xx} = -18x$ $\quad D = (-18x)(-4x) - (-4y)^2$
$f_{xy} = -4y$ $\quad D = 72x^2 - 16y^2$
$f_{yy} = -4x$

$D(0,\sqrt{2}) = -32 < 0 \rightarrow (0, \sqrt{2})$ is a saddle point
$D(0,-\sqrt{2}) = -32 < 0 \rightarrow (0, -\sqrt{2})$ is a saddle point
$D(2/3, 0) = 32 > 0$ + $f_{xx}(2/3, 0) < 0$ so $(2/3, 0)$ is a local max
$D(-2/3, 0) = 32 > 0$ + $f_{xx}(-2/3, 0) > 0$ so $(-2/3, 0)$ is a local min

(13) $f(x,y) = x^4 + y^4 - 4xy$

$f_x = 4x^3 - 4y = 0$
$\frac{-4y}{-4} = \frac{-4x^3}{-4}$
$y = x^3$

$f_y = 4y^3 - 4x = 0$
$4x^9 - 4x = 0$
$4x(x^8 - 1) = 0$
$x = 0,\ x = 1, -1$

Critical Points: $(0,0), (1,1), (-1,-1)$

$f_{xx} = 12x^2 \quad f_{xy} = -4 \quad D = (12x^2)(12y^2) - (-4)^2$
$f_{yy} = 12y^2$

$D(0,0) = -16 < 0 \rightarrow (0,0) \rightarrow$ Saddle Point
$D(1,1) = 128 > 0$ + $f_{xx}(1,1) > 0$ so $(1,1)$ is a local Min.
$D(-1,-1) = 128 > 0$ + $f_{xx}(-1,-1) > 0$ so $(-1,-1)$ is a local min.

(19) $f(x,y) = \ln x + 2\ln y - x - 4y$

$f_x = \frac{1}{x} - 1 = 0$ $f_y = \frac{2}{y} - 4$ $y = 1/2$

$x = 1$

Critical Point $(1, 1/2)$

$f_{xx} = \frac{-1}{x^2}$

$f_{xy} = 0$

$f_{yy} = \frac{-2}{y^2}$

$D = \left(\frac{-1}{x^2}\right)\left(\frac{-2}{y^2}\right) - (0)^2$

$D(1, \tfrac{1}{2}) = -1\left(\frac{-2}{\frac{1}{4}}\right) = -1 \cdot -8 = 8 > 0$

and $f_{xx}(1, \tfrac{1}{2}) < 0$

so $(1, \tfrac{1}{2})$ is a local max

(21) $f(x,y) = x - y^2 - \ln(x+y)$

$f_x = 1 - \frac{1}{x+y} = 0$ $f_y = -2y - \frac{1}{x+y} = 0$

$x + y = 1$

$y = 1 - x$

$-2(1-x) - \frac{1}{x+1-x} = 0$

$-2x - 2 - \frac{1}{1} = 0$

$2x - 3 = 0$

$2x = 3$

$x = 3/2$

$y = -1/2$

Critical Point: $(3/2, -1/2)$

$f_{xx} = \frac{1}{(x+y)^2}$

$f_{yy} = -2 + \frac{1}{(x+y)^2}$

$f_{xy} = \frac{1}{(x+y)^2}$

$D(3/2, -1/2) < 0$ so $\boxed{(3/2, -1/2) \text{ is a saddle point}}$

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14.7

Rahul Paliga

14.7 - #23, 29, 35:

$u = x + 3y$ $\quad v = e^{y-x^2}$

$u' = 1$ $\quad v' = e^{y-x^2} \cdot -2x$

(23) ~~f~~$(x,y) = (x+3y)e^{y-x^2}$

$f_x = (x+3y)(-2xe^{y-x^2}) + e^{y-x^2} \rightarrow e^{y-x^2}(1-2x^2-6xy) = 0$

$1-2(-3y-3)^2-6(y)(-3y-3) = 0$

$y = -17/18$

$f_y = (x+3y)e^{y-x^2} + 3e^{y-x^2} \rightarrow e^{y-x^2}(x+3y+3) = 0$

$x + 3y + 3 = 0$

$x = -3y - 3$

$x = -1/6$

Critical Point: $(-1/6, -17/18)$

$f_{xx} = (2x^3 + 6x^2 - 3x - 3y)2e^{y-x^2}$

$f_{xy} = (1 - 6xy - 2x^2 - 6x)e^{y-x^2}$

$f_{yy} = (6 + x + 3y)e^{y-x^2}$

$D(-1/6, -17/18) > 0$ & $f_{xx}(-1/6, -17/18) > 0$ thus $(-1/6, -17/18)$ is a local min.

(29) $f(x,y) = x + y \quad 0 \le x \le 1, \; 0 \le y \le 1$

$f_x = 1$

$f_y = 1$ One solution $(1,1)$ $\quad f(1,1) = 2$

left side $x = 0 \quad 0 \le y \le 1$

$f(0,y) = y \quad F'(y) = 1$

$F(1) = 1 \quad F(0) = 0 \quad F(1) = 1$

Abs. Min on left side: 0

Abs. Max on left side: 1

Right Side: $x = 1, \; 0 \le y \le 1$

$f(1,y) = 1 + y \quad F'(y) = 1$

$F(0) = 1 \quad F(1) = 2$

Abs. Min = 1 $\quad$ Abs. Max = 2

Down Side: $y = 0; \; 0 \le x \le 1$

$F(x,0) = x \quad f'(x) = 1$

$F(0) = 0 \quad F(1) = 1$

Abs. Min = 0

Abs. Max = 1

Up Side: $y = 1 \; 0 \le x \le 1$

$F(x,1) = x + 1$

$F'(x) = 1 \quad F(0) = 1$

$F(1) = 2$ Abs. Min = 1

Abs. Max = 2

Overall:

Abs. Min = 0

Abs. Max = 2

(35) $f(x,y) = x + y - x^2 - y^2 - xy$ $\quad 0 \le x \le 2;\ 0 \le y \le 2$

$F_x = 1 - 2x - y = 0 \quad F_y = 1 - 2y - x$

$y = 1 - 2x \quad 1 - 2(1-2x) - x$

$1 - 2 + 4x - x = 0 \quad -1 + 3x = 0$

$y = \frac{1}{3} \quad x = \frac{1}{3}$

Critical Point $(\frac{1}{3}, \frac{1}{3}) \quad f(\frac{1}{3}, \frac{1}{3}) = \frac{1}{3}$

Left side $x = 0;\ 0 \le y \le 2$

$F(0,y) = y - y^2 \quad F'(y) = 1 - 2y \quad y = \frac{1}{2}$

$F(\frac{1}{2}) = \frac{1}{4} \quad F(0) = 0 \quad F(2) = -2$

Abs. Min. of left side $= -2$

Abs. Max of left side $= \frac{1}{4}$

Right Side: $x = 2 \quad 0 \le y \le 2$

$F(2,y) = -2 - y^2 - 2y \quad F'(y) = -2y - 2 \quad y = -1$

$F(-1) = -1 \quad F(0) = -2 \quad F(2) = -2$

Abs. Min of R.S. $= -2$ Abs. Max of R.S. $= -1$

Down Side: $0 \le x \le 2 \quad y = 0$

$F(x,0) = x - x^2 \quad F'(x) = 1 - 2x \quad x = \frac{1}{2} \quad F(\frac{1}{2}) = \frac{1}{4}$

$F(0) = 0 \quad F(2) = -2$

Abs. Min of D.S. at -2

Abs. Max of D.S. at $\frac{1}{4}$

Up Side: $0 \le x \le 2 \quad y = 2$

$F(x,2) = x + 2 - x^2 - 4 - 2x$

$F'(x) = 1 - 2x - 2 = 0 \quad x = -\frac{1}{2} \quad F(-\frac{1}{2}) = -\frac{7}{4}$

$F(0) = -2 \quad F(2) = -12$

Abs. Min of U.S. at -12

Abs. Max of U.S. at $-\frac{7}{4}$

Overall: Abs. Max at $\frac{1}{3}$

Abs. Min at -12