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October 10, 2020

14.6: #'s 1, 3, 5, 7, 15, ~~17~~, 23, 27, 29, 31

(1) $f(x,y,z) = x^2y^3 + z^4 \quad x = s^2,\ y = st^2 \quad z = s^2t$

a. $\frac{df}{dz} = 2xy^3 \quad \frac{df}{dy} = 3y^2x^2 \quad \frac{df}{dz} = 4z^3$

b. $\frac{dx}{ds} = 2s \quad \frac{dy}{ds} = t^2 \quad \frac{dz}{ds} = 2st$

c. $\frac{df}{ds} = 2xy^3(2s) + 3y^2x^2(t^2) + 4z^3(2st)$

(3) $f(x,y,z) = xy + z^2 \quad x = s^2 \quad y = 2rs \quad z = r^2$

$\frac{df}{ds} \Rightarrow \quad \frac{df}{dx} = y \quad \frac{dx}{ds} = 2s \Rightarrow \frac{df}{ds} = 2ys + 2rx$
$= 2(2rs)s + 2r(s^2)$
$= 4rs^2 + 2rs^2$
$= 6rs^2$

$\frac{df}{dy} = x \quad \frac{dy}{ds} = 2r$

$\frac{df}{dz} = 2z \quad \frac{dz}{ds} = 0$

* watch out if they ask for it in terms of the independent variables or not!

$\frac{df}{dr} = \quad \frac{df}{dx} = y \quad \frac{dx}{dr} = 0$

$\frac{df}{dy} = x \quad \frac{dy}{dr} = 2s$

$\frac{dz}{dy} = 2z \quad \frac{dz}{dr} = 2r$

$\rightarrow \frac{df}{dr} = y(0) + 2sx + 4zr$
$= 2sx + 4zr$
$= 2s(s^2) + 4(r^2)r$
$= 2s^3 + 4r^3$

(5) $\frac{\partial g}{\partial u}$ and $\frac{\partial g}{\partial v} \quad g(x,y) = \cos(x-y) \quad x = 3u - 5v \quad y = -7u + 15v$

$\frac{\partial g}{\partial x} = -\sin(x-y) \quad \frac{\partial x}{\partial u} = 3 \quad \frac{\partial y}{\partial u} = -7 \quad \therefore \frac{\partial g}{\partial u} = -3\sin(x-y) - 7\sin(x-y)$
$= -10\sin(10u - 20v)$

$\frac{\partial g}{\partial y} = \sin(x-y) \quad \frac{\partial x}{\partial v} = -5 \quad \frac{\partial x}{\partial v} = 15 \quad \therefore \frac{\partial g}{\partial v} = 20\sin(10u - 20v)$

(7) $\frac{\partial F}{\partial y} \quad F(u,v) = e^{u+v} \quad u = x^2 \quad v = xy$

$\frac{\partial F}{\partial u} = e^{u+v} \quad \frac{\partial u}{\partial y} = 0 \quad \frac{\partial v}{\partial y} = x \quad \therefore \frac{\partial F}{\partial u} = xe^{u+v}$

(15) $\frac{\partial g}{\partial u}$ at $(u,v) = (0,1)$ where $g(x,y) = x^2 - y^2$ $x = e^u\cos v$ $y = e^u\sin v$

? $\frac{\partial g}{\partial x} = 2x \quad \frac{\partial x}{\partial u} = e^u\cos v \quad \frac{\partial g}{\partial u} = 2xe^u\cos v - 2ye^u\sin v$
$= -2e$

$\frac{\partial g}{\partial y} = -2y \quad \frac{\partial y}{\partial u} = e^u\sin v$

(23) $x = s + t \quad y = s - t \quad f(x,y) = 2x^2 + xy$

$\frac{\partial f}{\partial x} = 4x \quad \frac{\partial f}{\partial y} = y$

$\frac{\partial f}{\partial s} = 4x + y \quad \frac{\partial f}{\partial t} = 4x - y$

$\rightarrow (4x)^2 - y^2 = (4x)^2 - (y)^2$ ✓

works $\Rightarrow \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2 = \frac{\partial f}{\partial s}\frac{\partial f}{\partial t}$

(27) $\frac{\partial z}{\partial x} \quad x^2y + y^2z + xz^2 = 0$

$2xy + z'y^2 + z^2 + 2zz'x = 0$
$z'(y^2 + 2zx) = -2xy + z^2$
$z' = \frac{-2xy + z^2}{y^2 + 2zx}$

(29) $\frac{\partial z}{\partial y} \quad e^{xy} + \sin(xz) + y = 0$

$xe^{xy} + z'\cos(xz) + 1 = 0$

***!
$z' = \frac{-(1 + xe^{xy})}{x\cos(xz)}$

(31) $\frac{\partial w}{\partial y} \quad \frac{1}{w^2+x^2} + \frac{1}{w^2+y^2} = 1 \quad (x,y,w)$ @ $(1,1,1)$

$= \frac{2ww'}{(w^2+x^2)^2} + \frac{2ww'2y}{(w^2+y^2)^2} = 0$

@ $(1,1,1) = \frac{2w'}{4} + \frac{4w'}{4} = 0$

$w'\left(\frac{2}{4} + 1\right) = 0$

$\frac{1}{2}w' = 1$

$w' = -\frac{1}{2}$

14.7: 1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35 10/10/20

① $P=(a,b)$ $f(x,y)=x^2+y^4-4xy$

a. $f_x(x,y)=0$
$f_x=2x-4y$
$f_x(a,b)=2a-4b$
$2a=4b$
$a=2b$ ✓

$f_y=4y^3-4x$
$f_y(x,y)=0$
$4y^3-4x=0$
$y^3-x=0$
$y=0$ $x=0$ ↑
$P(0,0)$

b. $f_{xx}=2$
$f_{yy}=12y^2-4$
$f_{xy}=-4$
→ $D=24y^2-8-16$
$=-8$
negative so $(0,0)$ is a saddle point

↑?

③ $f(x,y)=8y^4+x^2+xy-3y^2-y^3$
* how to read contour maps
$f_x=2x+y$
$f_{xx}=2$
$f_y=32y^3+x-6y-3y^2$
$f_{yy}=96y^2-6-6y$
$f_{xy}=1$
$2(96y^2-70)-1$
$192y^2-141=D$

$(0,0)$ = saddle point
$\left(\frac{13}{64},\frac{-13}{32}\right)$ and $\left(-\frac{1}{4},\frac{1}{2}\right)$ are local minima
I thought only one type can exist?

⑤ $y^2-2xy+y$ $2xy-x^2+x=0$
$y(y-2x+1)=0$ $x(2y-x+1)=0$ satisfy
* showing that the critical points (x,y) should satisfy $f_x=0$ and $f_y=0$

a. *** critical points $\Rightarrow y(y-2x+1)=0$
$f_x=y^2-2xy+y=0$
$f_y=2xy-x^2+x=0$ $\Rightarrow x(2y-x+1)=0$ ⇒ $x=0$ $y=0$

b. $y=0$ or $y-2x+1=0 \rightarrow y=2x-1$
$x(2(0)-x+1)=0$
$x(-x+1)=0$ $\Rightarrow x=0$ or $x=1$
→ so $(0,0)$ and $(0,1)$

$x(2(2x-1)-x+1)=0$
$x(4x-2-x+1)=0$
$x(3x-1)=0$ $\Rightarrow x=0$ and $\frac{1}{3}$
→ so, $(0,-1)$ and $\left(\frac{1}{3},-\frac{1}{3}\right)$

For $x=0$ $\Rightarrow y=2(0)-1=-1$
For $x=\frac{1}{3}$ $\Rightarrow y=2\left(\frac{1}{3}\right)-1=-\frac{1}{3}$

c. Using the 2nd Deriv. test to determine the nature of the critical points.
$f_{xx}=-2y$
$f_{yy}=2x$
$f_{xy}=2y-2x+1$
$D(x,y)=-4xy-(2y-2x+1)^2$
D @ $(0,0) \Rightarrow -1<0$
$(0,1) \Rightarrow -1<0$
$(0,-1) \Rightarrow -1<0$
$\left(\frac{1}{3},-\frac{1}{3}\right) \Rightarrow \frac{2}{3}>0$
∴ So, @ $(0,0)$, $(0,1)$, $(0,-1)$ are saddle points.
@ $\left(\frac{1}{3},-\frac{1}{3}\right)$ the function has a local minimum.

***!

⑦ $f(x,y)=x^2+y^2-xy+x$
? $f_x=2x-y+1=0$
$2(2x-y=-1)$ $\Rightarrow 4x-2y=-2$
$f_y=2y-x=0$
$-x+2y=0$
$3x=-2$
$x=-\frac{2}{3}$ $y=-\frac{1}{3}$
$f_{xx}=2$
$f_{yy}=2$
$f_{xy}=-1$
→ $D=4-1$
$D=3>0$
↳* $\left(-\frac{2}{3},-\frac{1}{3}\right)$ is a local minimum.

⑪ $f(x,y)=4x-3x^3-2xy^2$
$f_x=4-9x^2-2y^2=0$
$f_{xx}=-18x=0$ $f_{xy}=-4y$
$f_y=-4xy$
$f_{yy}=-4x$
$D=72xy-16y^2$
$4-9x^2-2y^2=0$
$-4xy=0$ $\Rightarrow$ $x=\frac{y-2}{3}$
$-x^2=\frac{-y^2-4}{2(4)}$
$-4xy=0$
$-4y+8(y)$ over 3 $=0$
$-4y+8y=0$
$-12y=0$
$(0,\pm\sqrt{2})$ saddle point
$\left(\frac{2}{3},0\right)$ local max
$\left(-\frac{2}{3},0\right)$ local min

⑬ $f(x,y)=x^4+y^4-4xy$
$f_x=4x^3-4y=0$ $\Rightarrow x^3-y=0$
$f_y=4y^3-4x=0$ $\Rightarrow y^3-x=0$
$(0,0)$, $(1,1)$ and $(-1,-1)$
* $D(x,y)=12x^2(12y^2)-16$
$D(0,0)=-16<0$ Saddle point
$D(1,1)=144-16>0$ local min.
$D(-1,-1)=144-16>0$ local min.

⑲ $f(x,y)=\ln x+2\ln y-x-4y$
$f_x=\frac{1}{x}-1$ $f_{xx}=-\frac{2}{x^2}$ $f_{xy}=0$
$f_y=\frac{2}{y}-4$ $f_{yy}=-\frac{4}{y^2}$
$D=\frac{8}{x^2y^2}$ $x=1$ $y=\frac{1}{2}$
$D\left(1,\frac{1}{2}\right)=16>0$ local max.
$f_x=1-\frac{1}{x+y}$, $f_{xy}=\frac{1}{(x+y)^2}$
$f_y=-2y-\frac{1}{x+y}$

㉑ $f(x,y)=x-y^2-\ln(x+y)$
$x=\frac{3}{2}$ $y=-\frac{1}{2}$ $\Rightarrow D\left(\frac{3}{2},-\frac{1}{2}\right)<0$ saddle point

㉓ $f(x,y)=(x+3y)e^{y-x^2}$
↳ $D\left(-\frac{1}{6},-\frac{17}{18}\right)>0$ local min
$0\le x\le 1$ $0\le y\le 1$

㉙ $f(x,y)=x+y$
$f_x=1$ $f_y=1$ no sol'n so no critical points
? ↗ $F(0,y)=y$ $F'(y)=1$

㉟ $f(x,y)=x+y-x^2-y^2-xy+1$
$f(x,y)\Rightarrow\left(\frac{1}{3},\frac{1}{3}\right)$
$x=0$ $\Rightarrow F(0,y)=y-y^2$
$F\left(\frac{1}{2}\right)=\frac{1}{4}$
$F(2)=-2$
$F(0)=0$
$x=2$ $\Rightarrow F(2,y)=2+y-4-y^2-2y$
$=-y-y^2-2$
$y=0$ $\Rightarrow F(x,0)=x-x^2$ $x=0,1$
$F'(0)=1$
$F'(1)=-1$
$F(2)=-3$
$y=2 \Rightarrow F(x,2)=x+2-x^2$
$=-x-2-x^2$
$=-x^2-x-2$