14.6: 1 3 5 7 15 opt. 17 23 22 29 31
14.7: 1 3 5 7 11 13 19 21 23 29 35

Chapter 14 HW

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14.6

1) $f(x,y,z) = x^2y^3 + z^4 \quad x = s^2 \quad y = st^2 \quad z = s^2t$

a) $\frac{\partial f}{\partial x} = 2xy^3 \quad \frac{\partial f}{\partial y} = 3x^2y^2 \quad \frac{\partial f}{\partial z} = 4z^3$

b) $\frac{dx}{ds} = 2s \quad \frac{\partial y}{\partial s} = t^2 \quad \frac{\partial z}{\partial s} = 2st$

c) $\frac{\partial f}{\partial z} = 4(s^2)(st^2)^3 s + 3(s^2)^2(st^2)^2 t^2 + 8(s^2t)^3 st$

3) $f(x,y,z) = xy + z^2 \quad x = s^2 \quad y = 2rs \quad z = r^2$

$\partial s/ \quad \backslash \partial r \quad \partial s/ \quad \backslash \partial r \quad \partial s/ \quad \backslash \partial r$

$2s \quad 0 \quad 2r \quad 2s \quad 0 \quad 2r$

$$\frac{\partial f}{\partial s} = \boxed{2rs \cdot 2s + s^2 \cdot 2r} + \cancel{2r^2 \cdot 0}$$

$$\frac{\partial f}{\partial r} = \cancel{2rs \cdot 0} + \boxed{s^2 \cdot 2s + 2r^s\, 2r}$$

5) $g(x,y) = \cos(x-y) \quad x = 3u - 5v \quad y = -7u + 15v$

$-\sin(x-y)\cdot 1$

$+\sin(x-y)(+1)$

$\partial u/ \quad \backslash \partial v \quad \partial u/ \quad \backslash \partial v$

$3 \quad -5 \quad -7 \quad 15$

$$\boxed{\frac{\partial g}{\partial u} = -3\sin(10u-20v) - 7\sin(10u-20v)}$$

$$\boxed{\frac{\partial g}{\partial v} = 5\sin(10u-20v) + 15\sin(10u-20v)}$$

14.6 Cont

7) $F(u,v) = e^{u+v}$ $\quad u = x^2 \quad v = xy$

e^{u+v}

e^{u+v}

$2x \quad 0 \qquad y \quad x$

$$\boxed{\frac{\partial F}{\partial y} = e^{x^2+xy} \cdot x}$$

15) $g(x,y) = x^2 - y^2$; $x = e^u\cos(v)$; $y = e^u\sin(v)$; $(u,v) = (0,1)$

$-2y$

$2x$

$e^u\cos(v) \quad -e^u\sin(v) \qquad e^u\sin(v) \quad e^u\cos(v)$

$x(0,1) = \cos(1)$

$y(0,1) = \sin(1)$

$x_u(0,1) = \cos(1) \quad y_u = \sin(1)$

~~$x_v(0,1) = \sin(1) \quad y_v = \cos(1)$~~

$$\boxed{\frac{\partial g}{\partial u} = -2(\sin(1))\cdot\cos(1) + 2(\cos(1))(\sin(1))}$$

23) $x = s+t \quad y = s-t$

Prove that $\left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2 = \boxed{\frac{\partial f}{\partial s}\frac{\partial f}{\partial t}}$

$$= \left(\frac{\partial f}{\partial x}\cdot\underbrace{\frac{\partial x}{\partial s}}_{1} + \frac{\partial f}{\partial y}\cdot\underbrace{\frac{\partial y}{\partial s}}_{1}\right)\left(\frac{\partial f}{\partial x}\underbrace{\frac{\partial x}{\partial t}}_{1} + \frac{\partial f}{\partial y}\underbrace{\frac{\partial y}{\partial t}}_{-1}\right)$$

$$= \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right)\left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}\right)$$

$$= \boxed{\left(\frac{\partial f}{\partial x}\right)^2 + \cancel{\left(\frac{\partial f}{\partial x}\frac{\partial f}{\partial y}\right)} - \cancel{\left(\frac{\partial f}{\partial x}\cdot\frac{\partial f}{\partial y}\right)} - \left(\frac{\partial f}{\partial y}\right)^2}$$

14.6 Cont

27) $\left(x^2y + y^2z + xz^2 = 10\right)'$ find $\frac{\partial z}{\partial x}$

w.r.t. x: $2xy + y^2z' + (z^2 + 2xzz') = 0$

$z'(y^2 + 2xz) = -z^2 - 2xy$ $\boxed{\frac{\partial z}{\partial x} = \frac{-z^2 - 2xy}{y^2 + 2xz}}$

29) $\left(e^{xy} + \sin(xz) + y = 0\right)'$ find $\frac{\partial z}{\partial y}$

w.r.t. y: $xe^{xy} + xz'\cos(xz) + 1 = 0$

$\boxed{z' = \frac{-xe^{xy} - 1}{x\cos(xy}}$

31) $\left(\frac{1}{w^2 + x^2} + \frac{1}{(w^2 + y^2)} = 1\right)'$ find $\frac{\partial w}{\partial y}$ @ $(x, y, w) = (1, 1, 1)$

w.r.t. y: $\frac{-(2ww')}{(w^2+x^2)^2} - \frac{(2ww' + 2y)}{(w^2+y^2)^2} = 0$

$\cancel{2}ww'\left(\frac{1}{(w^2+x^2)^2} + \frac{1}{(w^2+y^2)^2}\right) = \frac{-\cancel{2}y}{(w^2+y^2)^2}$

$\boxed{w' = \frac{y}{w(w^2+y^2)^2} \cdot \left(\frac{1}{(w^2+x^2)^2} + \frac{1}{(w^2+y^2)^2}\right)^{-1}}$

@ $(1, 1, 1) \Rightarrow$

$= \frac{1}{4} \cdot 2 = \boxed{\frac{1}{2}}$ ☆

14.7

1) $P = (a, b)$ is crit pt of $f(x,y) = x^2 + y^4 - 4xy$

$f_x = 2x + 0 - 4y = 0 \Rightarrow x - 2y = 0 \quad \boxed{x = 2y}$

$f_y = 4y^3 - 4x = 0 \Rightarrow y^3 - 2y = 0 \Rightarrow y(y^2 - 2) = 0$

$(y + \sqrt{2})(y - \sqrt{2})$

a) Soln @ $\boxed{\begin{array}{ll} x = 0 & y = 0 \\ x = 2\sqrt{2} & y = \sqrt{2} \\ x = -2\sqrt{2} & y = -\sqrt{2} \end{array}}$

b) $(0, 0, 0)$ is a saddle pt

$(2\sqrt{2}, \sqrt{2}, -4)$ is an absolute min

$(-2\sqrt{2}, -\sqrt{2}, -4)$ is an abs min

3) $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_x = 2x + y = 0 \Rightarrow \boxed{x = \frac{-y}{2}}$

$f_y = 32y^3 + x - 6y - 3y^2 \Rightarrow 32y^3 - 3y^2 - \frac{13}{2}y = 0$

$\boxed{y = 0} \Rightarrow$

$32y^2 - 3y - \frac{13}{2} = 0 \qquad y = \frac{3 \pm \sqrt{9 - (4(32)(-\frac{13}{2}))}}{64} = \boxed{\frac{3 \pm 29}{64}}$

$\boxed{y = -\frac{13}{32}, 0, \frac{1}{2}} \quad \boxed{x = \frac{13}{64}, 0, \frac{-1}{4}}$

14.7 Cont

3) $P_1 = \left(\frac{+13}{64}, \frac{-13}{32}\right)$ $P_2 = (0, 0)$ $P_3 = \left(\frac{-1}{4}, \frac{1}{2}\right)$

Based on contour map

P_1 is local min
P_2 is saddle pt
P_3 is local min

5) $f(x, y) = y^2x - yx^2 + xy$

a) $\boxed{\begin{array}{l} f_x = y(y - 2x + 1) = 0 \\ f_y = x(2y - x + 1) = 0 \end{array}}$ $\Rightarrow$ $y = 0$ $y - 2x + 1 = 0$ $y = 2x - 1$

$x = 0$ $2y - x + 1 = 0$ $4x - 2 - x + 1$

$3x - 1 = 0$

$x = \frac{1}{3}$

b) $\boxed{@\ x = 0,\ y - 2(0) + 1 = 0,\ y = -1}$ ✓

@ $x = 0$, $2y - (0) + 1 = 0$, $y = \frac{-1}{2}$ X $\boxed{@\ x = 0,\ y = 0}$ ✓

~~@ $y = 0$, $0 - 2x + 1 = 0$, $x = \frac{1}{2}$~~ X

$\boxed{@\ y = 0,\ 2(0) - x + 1 = 0,\ x = 1}$ ✓

$\boxed{@\ x = \frac{1}{3}, \ldots\ y = \frac{-1}{3}}$ ✓

c) $P_1 = (0, 0)$ $P_2 = (0, -1)$ $P_3 = (1, 0)$ $P_4 = \left(\frac{1}{3}, \frac{-1}{3}\right)$

$f_{xx} = -2y$ $f_{yy} = 2x$ $f_{xy} = 2y - 2x + 1$

14.7 Cont

5) $D_1 = 0\cdot 0 - (0-0+1)^2 = -1$ $\quad$ P_1: Saddle Pt

$D_2 = (-2(-1))(0) - (2(-1)-0+1)^2 < 0$ $\quad$ P_2: Saddle Pt

$D_3 = (0\cdot(\sim)) - (2(0)-2(1)+1)^2 < 0$ $\quad$ P_3: Saddle Pt

$D_4 = \left(\frac{2}{3}\cdot\frac{2}{3}\right) - \left(2\left(\frac{-1}{3}\right) - 2\left(\frac{1}{3}\right)+1\right)^2$

$\frac{4}{9} - \left(\frac{-4}{3}+1\right)^2 = \frac{3}{9} = \frac{1}{3} > 0,\ f_{xx} > 0 \therefore$

P_4 is local Min

7) $f(x,y) = x^2 + y^2 - xy + x$

$f_x = 2x - y + 1 = 0 \Rightarrow 4y - y + 1 = 0$ $\quad$ $\boxed{y = \frac{1}{3}}$

$f_y = 2y - x = 0 \quad x = 2y$ $\quad$ $\boxed{x = \frac{2}{3}}$

$f_{xx} = 2 \quad f_{yy} = 2$

$f_{xy} = -1$ $\quad$ $D = 4 - (-1)^2 = 3 > 0;$

$f_{xx} > 2;$

$\therefore P = \left(\frac{2}{3}, \frac{1}{3}\right)$ is a local min

11) $f(x,y) = 4x - 3x^3 - 2xy^2$

$f_x = 4 - 9x^2 - 2y^2 = -9x^2 + 4 - 2y^2 = 0$

$f_y = -4xy = 0$

14.7 Cont

11) $f_x \overset{@\,y=0}{=} -9x^2+4=0 \Rightarrow (-3x+2)(3x+2) \Rightarrow \boxed{x=\frac{2}{3}, \frac{-2}{3}}$

$f_x \overset{@\,x=0}{=} 4-2y^2=0 \Rightarrow (2+\sqrt{2}y)(2-\sqrt{2}y) \Rightarrow \boxed{y=\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}}$

$P_1=(\frac{2}{3}, 0)$	$P_2=(\frac{-2}{3}, 0)$
$P_3=(0, \frac{1}{\sqrt{2}})$	$P_4=(0, \frac{-1}{\sqrt{2}})$

$\begin{bmatrix} f_{xx}=-18x & f_{xy}=-4y \\ -4y & f_{yy}=-4x \end{bmatrix}$ $D_i=+72(x_i \cdot y_i)-(16y_i^2)$

$D_1=+72(0)-(16(0)^2)=0$ Inconclusive for P_1

$D_2= \quad . \quad . \quad . \quad =0$ Inconclusive for P_2

$D_3=72(0)-(16(\frac{1}{2}))<0$ P_3 is a saddle Pt

$D_4=72(0)-(16(\frac{1}{2}))<0$ P_4 is a saddle Pt

14.7 Cont

13) $f(x, y) = x^4 + y^4 - 4xy$

$f_x = 4x^3 - 4y = 0 \Rightarrow x^3 = y \quad x = \sqrt[3]{y}$

$f_y = -4y^3 - 4x = 0 \Rightarrow y^3 = -x = -\sqrt[3]{y}$

$P = \boxed{\begin{matrix} x = 0 \\ y = 0 \end{matrix}}$ $\qquad y^9 = -y \Rightarrow y^9 + y = 0$

$y(y^8 + 1) = 0$

$\boxed{y = 0}$ $\qquad y = \pm\sqrt[8]{-1}$???

$$\begin{bmatrix} f_{xx} = 12x^2 & f_{xy} = -4 \\ -4 & f_{yy} = -12y^2 \end{bmatrix}$$

$D = -144(0 \cdot 0) - (16) = -16 \therefore$

$\boxed{P = (0, 0) \text{ is a saddle point}}$

19) $f(x, y) = \ln(x) + 2\ln(y) - x - 4y$

$f_x = \frac{1}{x} - 1 = 0 \quad x = 1$

$f_x = \frac{2}{y} - 4 = 0 \quad y = \frac{1}{2}$ $\qquad P = (1, \frac{1}{2})$

$D = \frac{2}{x^2 y^2} - 0 = \frac{2}{\frac{1}{4}} = 8 > 0$

$f_{xx} = -\frac{1}{x^2} \qquad f_{xy} = 0$

$f_{xx}(1, \frac{1}{2}) = -1 < 0$

$0 \qquad f_{yy} = \frac{-2}{y^2}$

$\boxed{P = (1, \frac{1}{2}) \text{ is a local max}}$

14.7 Cont

21) $f(x,y) = x - y^2 - \ln(x+y)$

$f_x = 1 - \frac{1}{x+y} = 0$ $\quad \underbrace{x+y=1}$

$f_y = 2y - \frac{1}{x+y} = 0$ $\quad 2y(x+y) = 1$ $\quad 2y = 1,$ $\boxed{y = \frac{1}{2}, x = \frac{1}{2}}$

$f_{xx} = \frac{1}{(x+y)^2}$ $\quad f_{xy} = \frac{1}{(x+y)^2}$ $\quad P = (\frac{1}{2}, \frac{1}{2})$

$\frac{1}{(x+y)^2}$ $\quad f_{yy} = 2 + \frac{1}{(x+y)^2}$

$$D = \left(\frac{1}{1}\right)(2 + 1) - \left(\frac{1}{(1)^4}\right) = 2 > 0$$

$$f_{xx}\left(\frac{1}{2}, \frac{1}{2}\right) = 1 > 0 \therefore$$

P is a local min

23) $f(x,y) = (x+3y)e^{y-x^2}$

Never 0

$f_x = e^{y-x^2} + (x+3y)(-2x)e^{y-x^2} = \boxed{\cancel{e^{y-x^2}}}(-2x^2 - 6xy + 1) = 0$

$f_y = 3e^{y-x^2} + (x+3y)(1)e^{y-x^2} = \boxed{\cancel{e^{y-x^2}}}(x + 3y + 3) = 0$

$\boxed{y = \frac{-3-x}{3}}$

$-2x^2 - \cancel{6}^{2}x\left(\frac{-3-x}{\cancel{3}}\right) + 1 = 0$

$-2x^2 - 2x(-3-x) + 1 = 0$

$\cancel{-2x^2 + 2x^2} + 6x + 1 = 0$

$\boxed{x = \frac{-1}{6}}$

$y = \frac{-3 - \frac{1}{6}}{3} = \frac{\frac{-17}{6}}{3} = \boxed{\frac{-17}{2}}$

$\boxed{P = \left(\frac{1}{6}, \frac{-17}{2}\right)}$

14.7 Cont

29) $f(x,y) = x+y \quad 0 \le x \le 1 \quad 0 \le y \le 1$

Abs Max: $P(1,1)$ $f(1,1) = 2$

Abs Min: $P(0,0)$ $f(0,0) = 0$

35) $f(x,y) = x+y-x^2-y^2-xy \quad 0 \le x \le 2, \; 0 \le y \le 2$

a) $f_x = 1-2x-y = 0 \quad y = 1-2x$

$f_y = 1-2y-x = 0 \quad 1-2(1-2x)-x = 0 \quad 3x = 1 \quad \boxed{x = \frac{1}{3}}, \boxed{y = \frac{1}{3}}$

$\boxed{f\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{2}{3} - \frac{2}{9} - \frac{1}{9} = \frac{1}{3}}$

b) $f(x,0) = x - x^2 \qquad f(0,0) = 0$

$1 - 2x = 0$

$x = \frac{1}{2} \longrightarrow$

$\boxed{\begin{array}{l} f(2,0) = -2 \text{ Abs Min on Edge} \\ f(\frac{1}{2}, 0) = \frac{1}{2} \text{ Abs Max on Edge} \end{array}}$

c) $f(x,2) = x+2-x^2-4-2x = -x^2-x-2$

$f_x(x,2) = -2x-1 = 0 \quad \cancel{x = -\frac{1}{2}}$

$f(0,y) = y - y^2$

$f_y(0,y) = 1-2y \Rightarrow y = \frac{1}{2}$

f

$f(0,2) = -2$

$f(2,2) = -8$

$f(0,0) = 0$

$f(0, \frac{1}{2}) = \frac{1}{2}$

$f(0,2) = -2$

d) Abs Max on Square: $\boxed{f(0, \frac{1}{2}) = f(\frac{1}{2}, 0) = \frac{1}{2}}$

Abs Min on Square: $\boxed{f(2,2) = -8}$