homework 5

14.6

1.) $f(x,y,z) = x^2y^3 + z^4$

a.) $f_x = 2xy^3$, $f_y = 3x^2y^2$, $f_z = 4z^3$

$= (s^2)^2(st^2)^3 + (s^2t)^4$

$= \cancel{s^4 s^3 t^6 + s^8 t^4}$

$= \cancel{s^7 t^6 + s^8 t^4}$

$(s^2) = x$ $(st^2) = y$ $(s^2t) = z$

b.) $f_t = 2s$ $f_s = t^2$ $f_s = 2st$

$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$

c.) $= 2(s^2)(st^2)\cdot(2s) + 3(s^2)^2(st^2)^2\cdot(st^2) + 4(s^2t)^3\cdot(2st)$

$= 2(s^3t^2)\cdot(2s) + 3(s^4t^4)\cdot(st^2) + 4(s^6t^3)\cdot(2st)$

$= 7s^6t^6 + 8s^7t^4$

3.) $\frac{\partial f}{\partial s}, \frac{\partial f}{\partial r}$; $f(x,y,z) = xy + z^2$, $x = s^2$ $y = 2rs$ $z = r^2$

$y, x, 2z$

$2s, 2r, 0$ | $0, 2s, 2r$

$(2rs)\cdot 2s + (s^2)\cdot(2r) + 0$ | $0 + (s^2)\cdot(2s) + 2(r^2)\cdot(2r)$

$\frac{\partial f}{\partial s} = 6rs^2$ $\frac{\partial f}{\partial r} = 2s^3 + 4r^3$

7.) $\frac{\partial F}{\partial y}$, $F(u,v) = e^{u+v}$, $u = x^2$ $v = xy$

$\cancel{2x}$ | $0, x$

$\boxed{\frac{\partial F}{\partial y} = x \cdot e^{x^2+xy}}$

15.) $g(x,y) = x^2 - y^2$, $x = e^u \cos v$, $y = e^u \sin v$

$2x, -2y$

$2(e^u \cos v)\cdot e^u \cos v + -2(e^u \sin v)\cdot(e^u \sin v)$

$2e^{2u}\cos^2 v + -2e^{2u}\sin^2 v \Big|_{(u,v)=(0,1)} = 2(1)\cos 2 = \boxed{2\cos(2)}$

23.) $x = s+t$ $y = s-t$

$\frac{\partial}{\partial s}: 1, -1$

$\frac{\partial}{\partial t}: -1, 1$

$\frac{\partial}{s}\cdot\frac{\partial}{t} = -1$

not sure :

27.) $\frac{\partial z}{\partial x}$, $x^2y + y^2z + xz^2 = 10$

$10 : \cancel{\frac{\partial F}{\partial y}} \cancel{\partial F}$

$10 : \frac{\partial F}{\partial x} = \frac{\partial F}{\partial x}\frac{\partial x}{\partial x} \cancel{+ \frac{\partial F}{\partial x}\frac{\partial y}{\partial x}} + \frac{\partial F}{\partial x}\frac{\partial z}{\partial x}$

$= \frac{\partial F}{\partial x} \cdot 1 \cancel{+}$

$$\frac{\partial z}{\partial x} : \frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial z}} \quad \boxed{-\frac{z^2+y}{2xz+y^2}}$$

29.) $\frac{\partial z}{\partial y}$; $e^{xy} + \sin(xz) + y = 0$

$$\frac{\partial z}{\partial y} : -\frac{\frac{\partial F}{\partial y}}{\frac{\partial F}{\partial z}} \quad \boxed{: -\frac{xe^{xy}+1}{x\cos(xz)}}$$

31.) $\frac{1}{w^2+x^2} + \frac{1}{w^2+y^2} = 1$

$$\frac{\partial}{\partial w} : \frac{-\frac{2w}{(w^2+x^2)^2} - \frac{2w}{(w^2+y^2)^2}}{\frac{2y}{(w^2+y^2)^2}} = \left(\frac{-2w}{(w^2+x^2)^2} - \frac{2w}{(w^2+y^2)^2}\right) \cdot \frac{(w^2+y^2)^2}{2y}$$

$$\frac{-y(w^2+x^2)^2}{w((w^2+y^2)^2 + (w^2+x^2)^2)}$$

14.7

1.) $P(a,b)$ $f(x,y) = x^2 + y^4 - 4xy$

$f_x : 2x - 4y \rightarrow 2a - 4b : 2a : 4b \quad a = 2b$

$f_y : 4y^3 - 4x \rightarrow$ ~~AROA~~ $4y^3 - 4x : 0$

$P(0,0) \rightarrow$ saddle point

$P(2\sqrt{2}, \sqrt{2}) \rightarrow P(-2\sqrt{2}, -\sqrt{2})$ local minima

~~PyBM~~ f -4 is absolute minimum

3.) $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$(0,0)$ saddle point

$f_x : y + 2x$

$f_y : 32y^3 - 3y^2 - 6y + x$

~~tazar~~

5.) $f(x,y) = y^2x - yx^2 + xy$

a.) $f_x = y^2 - 2xy + y \rightarrow \quad y(y - 2x + 1) = 0$

$f_y = 2xy - x^2 + x \rightarrow \quad x(2y - x + 1) = 0$

b.) critical points $(0,0)$

$y = 0 \rightarrow \quad 0 - 2x + 1 = 0$

$x = 1 \quad (1,0)$

$x = 0 \quad 2y - 0 + 1 = \quad y = -1 \ (0,-1)$

7.) $f(x,y) = x^2 + y^2 - xy + x \quad f_x = 2x + 1 - y \rightarrow \left(-\frac{2}{3}, -\frac{1}{3}\right)$

$f_{xx} = 2 > 0 \rightarrow$ local minimum (

11.) $f(x,y) = 4x - 3x^3 - 2xy^2 \quad f_x = 4 - 9x^2 - 2y^2 = (0, \pm\sqrt{2})$ saddle

$f_{xx} = -18x < 0$ local maximum $\left(\frac{2}{3}, 0\right)$

$\left(-\frac{2}{3}, 0\right)$ local minima

19.) $f(x,y) = \ln x + 2\ln y - x - 4y$

$f_x = \frac{1}{x} - 1$

$f_{xx} = -\frac{1}{x^2} \quad \left(1, \frac{1}{2}\right) =$ local max

21.) $f(x,y) = x - y^2 - \ln(x^2 + y^2)$

$f_x = -\frac{2x}{x^2+y^2} + 1$

$f_{xx} = -\frac{2(y^2 - x^2)}{(x^2+y^2)^2} \longrightarrow \left(\frac{3}{2}, \frac{1}{2}\right)$ saddle point

23.) $f(x,y) = (x + 3y)e^{y - x^2}$

$f_x = e^{y-x^2} - 2e^{y-x^2}x(x+3y) = 0$

$f_{xx} = 4e^{y-x^2}x^3 + 12e^{y-x^2}yx^2 - 6e^{y-x^2}x - 6e^{y-x^2}x - 6e^{y-x^2}y$

$\left(-\frac{1}{6}, -\frac{17}{18}\right)$ local min!

29.) $f(x,y) = x + y, \quad 0 \le x \le 1, \quad 0 \le y \le 1$

$1 + 1 = 2$, global max

$0 + 0 = 0$, global min

35.) $f(x,y) = x + y - x^2 - y^2 - xy$

$\rightarrow$ no figure 23?

$2 + 2 - 4 - 4 - 4 = 4 - 12 = -8$?