Due Oct 04

HW due 10/5

12.3

1. a) $\frac{\partial z}{\partial x} = 2xy^3$

$\frac{\partial z}{\partial y} = 3x^2y^2$

$\frac{\partial z}{\partial t} = 4t^3$

b) $\frac{\partial z}{\partial x} = 2s$

$\frac{\partial z}{\partial y} = t^2$

$\frac{\partial z}{\partial t} = 2st$

c) $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial z}{\partial t}\frac{\partial t}{\partial s}$

$\frac{\partial z}{\partial s} = 2xy^3(1) + 3x^2y^2(t) + 4t^3(2s)$

$= 2(s+t)(s-t)^3 + 3(s+t)^2(s-t)^2t + 8t^3s$

$= 2(s^2-t^2)(s-t)^2 + 3(s^2+2st+t^2)(s-t)^2t + 8t^3s$

$= 2(s^2-t^2)^3 + 3(s^2+2st+t^2)(s^2-2st+t^2)t + 8t^3s$

$= 2(s^2-t^2)^3 + 3(s^2-t^2)^2t + 6s^2t^2 + 2t^4 + 8t^3s$

3) $F(x,y) = \ln(ax^2+by^2)^{-1}$

$= -1\ln(ax^2+by^2)$

$\frac{\partial F}{\partial x} = \frac{-2ax}{ax^2+by^2}$

$\frac{\partial F}{\partial x} = -2ax(ax^2+by^2)^{-1}$

5) $g(t,v) = \cos(3te^{-2v}) - (-7t+\sin v)^4$

$= \cos(3te^{-2v}) - (-7t+\sin v)^4$

$\frac{\partial g}{\partial t} = -\sin(3te^{-2v})(3te^{-2v}) - 4(-7t+\sin v)^3(-7)$

$\frac{\partial g}{\partial v} = -\sin(3te^{-2v})(3te^{-2v})(-2) - 4(-7t+\sin v)^3(\cos v)$

7) $f(x,y) = 6^{x+y}$

$\frac{\partial f}{\partial y} = 6^{x+y}\ln 6$

9) $g(x,t) = \frac{at^2}{2a\sqrt{\pi t}}e^{-x^2/4a^2t} - \frac{bt^2}{2b\sqrt{\pi t}}e^{-x^2/4b^2t}$

$= \frac{at^2}{2a\sqrt{\pi t}}e^{-x^2/4a^2t} - \frac{bt^2}{2b\sqrt{\pi t}}e^{-x^2/4b^2t}$

$\frac{\partial g}{\partial t}\Big|_{t=1} = 2.5\sqrt{\pi}e^{-x^2}$

11) $z = e^{ax+by}(a^2-b^2)$

$\frac{\partial z}{\partial x} = a^2e^{ax+by} - b^2e^{ax+by}$

$\frac{\partial z}{\partial y} = ab e^{ax+by} - ab e^{ax+by}$

$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}$

12) $x^2+y^2+z^2 = f(x^2+y^2-z^2)$

$2x + 2z\frac{\partial z}{\partial x} = f'(x^2+y^2-z^2)(2x - 2z\frac{\partial z}{\partial x})$

$2x + 2z\frac{\partial z}{\partial x} = 2xf' - 2zf'\frac{\partial z}{\partial x}$

$2x + 2zf'\frac{\partial z}{\partial x} + 2z\frac{\partial z}{\partial x} = 2xf'$

$2x + 2z\frac{\partial z}{\partial x}(f'+1) = 2xf'$

$\frac{\partial z}{\partial x} = \frac{2xf'-2x}{2z(f'+1)}$

$\frac{\partial z}{\partial x} = \frac{x(f'-1)}{z(f'+1)}$

13) e^{xy}

$e^{xy}\cdot\frac{\partial}{\partial x}(xy) + \frac{\partial}{\partial x}(z^3-3xyz) = 0$

$ye^{xy} + 3z^2\frac{\partial z}{\partial x} - 3yz - 3xy\frac{\partial z}{\partial x} = 0$

$\frac{\partial z}{\partial x} = -\frac{ye^{xy}-3yz}{3z^2-3xy}$

$\frac{\partial z}{\partial x} = \frac{3yz-ye^{xy}}{3z^2-3xy}$

31) $\frac{\partial w}{\partial r} = \frac{\partial}{\partial r}\left(\frac{1}{\sqrt{r^2+s^2+t^2}}\right) = -\frac{1}{2}(r^2+s^2+t^2)^{-3/2}(2r)$

$\frac{\partial w}{\partial r}\Big|_{(2,3,6)} = -\frac{2}{7^3} = -\frac{2}{343}$

12.4

2) Σ limit understood to equal sum

3) $f(x,y) = 2y^3 + x^2 + xy - 3y^2 - y^3$

$f_x = 0 + 2x + y \quad \Rightarrow y = -2x$

$f_y = 3(2y^2) + x - 6y - 3y^2$

$\Rightarrow 0 = 3x^2 + \frac{1}{2}x - 6y - 3y^2$

$y = 0$

$x = 0$

$S.P. = (0,0)$

$f_{xx} = 2$

$f_{xy} = 1$

$f_{yy} = 12y^2 - 6y - 6$

$D = (2)(-6) - 1^2 = -13$

$\Rightarrow D < 0$ and $f_{xx} > 0$ min

9) $f(x,y) = y^2x - ye^x + xy$

$f_x = y^2 - ye^x + y$

$f_y = 2xy - e^x + x$

$\Rightarrow y^2 - ye^x + y = 2xy - e^x + x$

$\Rightarrow y^2 - ye^x + y - 2xy + e^x - x = 0$

$\Rightarrow y^2 + (1-2x-e^x)y + (e^x - x) = 0$

$= \frac{-(1-2x-e^x) \pm \sqrt{(1-2x-e^x)^2 - 4(e^x-x)}}{2}$

$= \frac{2x+e^x-1 \pm \sqrt{4x^2+4xe^x-4x+e^{2x}-2e^x+1-4e^x+4x}}{2}$

$= \frac{2x+e^x-1 \pm \sqrt{4x^2+4xe^x+e^{2x}-6e^x+1}}{2}$

$\Rightarrow y = \frac{2x+e^x-1 \pm (2x+e^x-3)}{2}$

$\Rightarrow y = x+e^x-2 \quad \text{or} \quad y = -1$

$\Rightarrow 0 = x^2 - xe^x + 2x - x + e^x - 2 + x$

$\Rightarrow 0 = x^2 + 2x - 2$

$x = \frac{-2 \pm \sqrt{4+8}}{2} = -1 \pm \sqrt{3}$

$y = -1$

$C.P. = (-1+\sqrt{3}, -1), (-1-\sqrt{3}, -1)$

7) $f(x,y) = x^2y^2 - xy + r$

$f_x = 2xy^2 - y \quad f_{xx} = 2y^2$

$f_y = 2x^2y - x \quad f_{xy} = 4xy - 1$

$\Rightarrow 2xy^2 - y = 2x^2y - x$

$\Rightarrow x = 2x^2y - 2xy^2$

$\Rightarrow 0 = 2xy(x-y)$

$x = 0 \quad y = 0$

$y = x$

$\Rightarrow 0 = 2x^3 - 2x^3$

$0 = 0$

$C.P. = (0,0)$

11) $f(x,y) = x^2 - 2xy^2 - 2y^4$

$f_x = 2x - 2y^2 - 2x$

$f_y = -4xy - 8y^3$

$f_{xx} = -2y^2$

$f_{yy} = -4x - 24y^2$

$f_{xy} = -4y$

$\Rightarrow -4xy - 8y^3 = -2y^2$

$\Rightarrow 4xy + 8y^3 - 2y^2 = 0$

$\Rightarrow 2y(2x + 4y^2 - y) = 0$

$y = 0 \quad x = 0$

$2x + 4y^2 - y = 0$

$x = \frac{y - 4y^2}{2}$

$S.P. = (0,0)$

$L.Mac (-\frac{1}{4}, \frac{1}{8}), (\frac{1}{2}, \frac{1}{2})$

13) $f(x,y) = \ln(x + 2y) - x - 4y$

$f_x = \frac{1}{x+2y} - 1 \quad x = -2$

$f_y = \frac{2}{x+2y} - 4 \quad y = 2$

$f_{xx} = -\frac{1}{(x+2y)^2}$

$f_{xx} = -\frac{1}{16}$

$f_{yy} = -\frac{4}{(x+2y)^2}$

$f_{yy} = -\frac{1}{4}$

$f_{xy} = -\frac{2}{(x+2y)^2}$

$f_{xy} = -\frac{1}{8}$

$S.P. (0,0)$

$L.Max (-2, 2)$

21) $f(x,y) = (x^2+2y)e^{y-x^2}$

$f_x = (x^2+2y)(e^{y-x^2})(-2x) + 2xe^{y-x^2}$

$f_y = (x^2+2y)(e^{y-x^2}) + 2e^{y-x^2}$

23) $f(x,y) = 4x+y \qquad 0 \le x \le 2$

max $4x$

$\max = 8$

35) $f(x,y) = 2x+y-x^2-y^2-2xy$

a) $f_x = 2 - 2x - 2y = 0$

$f_y = 2 - 2y - 2x = 0$

$y = \frac{2}{2}$

$2 = 2 - 2x - \frac{2}{2}$

$\frac{2}{2} = 2 - 2x$

$2 = 4 - 4x$

$-2 = -4x$

$x = \frac{1}{2}$

$y = \frac{1}{2}$

$(x,y) = (\frac{1}{2}, \frac{1}{2})$

b) $f(x,y) = x-y$

$f_x = 1 - 2x$

$-1 = -2x$

$x = \frac{1}{2}$