14.7

1. a) $f_x(x,y) = 2x - 4y\ f_y(x,y) = 4y^3 - 4x$

$f_x(a,b) = 0 \rightarrow 2a - 4b = 0 \rightarrow a = 2b$

$f_y(x,y) = 0 \rightarrow 4b^3 - 8b = 0 \rightarrow b = 0\ or\ \pm\sqrt{2}$

b) $f_{xx}(x,y) = 2\ f_{yy}(x,y) = 12y^2\ f_{xy}(x,y) = 0$

$D_{(\pm 2\sqrt{2}, \pm\sqrt{2})} > 0, the\ (\pm 2\sqrt{2}, \pm\sqrt{2})\ are\ local\ maximum$

$D_{(0,0)} = 0, the\ (0,0)\ is\ the\ saddle\ point.$

$According\ to\ Figure\ 18, the\ absolute\ minimum\ is - 4.$

5. a) $f_x(x,y) = y^2 - 2xy + y\ \ f_y(x,y) = 2xy - x^2 + x$

b) $critical\ points: (0,0), (1,0), (0,-1)\ and\ \left(\frac{1}{3}, -\frac{1}{3}\right)$

c) $f_{xx}(x,y) = -2y\ f_{yy}(x,y) = 2x\ f_{xy}(x,y) = 2y - 2x + 1$

$D_{(0,0)} = D_{(1,0)} = D_{(0,-1)} = 0, D_{\left(\frac{1}{3}, -\frac{1}{3}\right)} > 0, f_{xx}(x,y) > 0$

$saddle\ points: (0,0), (1,0), (0,-1), and\ local\ minimum\ point\ is\ \left(\frac{1}{3}, -\frac{1}{3}\right).$

7. $f_x(x,y) = 2x - y - 1\ f_y(x,y) = 2y - x, the\ critical\ point\ is\ \left(\frac{2}{3}, \frac{1}{3}\right)$

$$f_{xx}\left(\frac{2}{3}, \frac{1}{3}\right) = 2\ f_{yy}\left(\frac{2}{3}, \frac{1}{3}\right) = 2\ f_{xy}\left(\frac{2}{3}, \frac{1}{3}\right) = -1, D > 0\ and\ f_{xx}\left(\frac{2}{3}, \frac{1}{3}\right) > 0$$

$So, \left(\frac{2}{3}, \frac{1}{3}\right) is\ local\ minimum.$

11. $f_x(x,y) = 4 - 9x^2 - 2y^2\ f_y(x,y) = -4xy, critical\ points: \left(\pm\frac{2}{3}, 0\right) and\ \left(0, \pm\sqrt{2}\right)$

$f_{xx}(x,y) = -18x\ f_{yy}(x,y) = -4x\ f_{xy}(x,y) = 0$

$So, \left(0, \pm\sqrt{2}\right) are\ saddle\ points, \left(\frac{2}{3}, 0\right) is\ local\ maximum, \left(-\frac{2}{3}, 0\right)\ is\ local\ minimum.$

13. $f_x(x,y) = 4x^3 - 4y\ f_y(x,y) = 4y^3 - 4x\ , critical\ points: (0,0)\ and\ (\pm 1, \pm 1)$

$f_{xx}(x,y) = 12x^2\ f_{yy}(x,y) = 12y^2\ f_{xy}(x,y) = -4$

$So, (0,0)\ is\ saddle\ points\ and\ (\pm 1, \pm 1)\ are\ local\ maximun$

17. $f_x(x,y) = \cos(x+y) + \sin(x)\ \ f_y(x,y) = \cos(x+y)\ , critical\ points: \left(i\pi, \left(j + \frac{1}{2}\right)\pi\right)$

$f_{xx}(x,y) = -\sin(x+y) + \cos(x)\ \ f_{yy}(x,y) = -\sin(x+y)\ \ f_{xy}(x,y) = -\sin(x+y)$

$i, j\ are\ even, saddle\ points;\ i\ and\ j\ are\ odd, local\ maximum; i\ is\ even, j\ are\ odd, local\ minimum$

$i\ is\ odd, j\ is\ even, saddle\ points$

19. $f_x(x,y) = \frac{1}{x} - 1\ f_y(x,y) = \frac{2}{y} - 4\ critical\ point: \left(1, \frac{1}{2}\right)$

$f_{xx}\left(1, \frac{1}{2}\right) = -1\ f_{yy}\left(1, \frac{1}{2}\right) = -8\ f_{xy}\left(1, \frac{1}{2}\right) = 0, \left(1, \frac{1}{2}\right)\ is\ local\ maximum.$

23. $f_x(x,y) = e^{y-x^2} - 2x(x+3y)e^{y-x^2}\ f_y(x,y) = 3e^{y-x^2} + y(x+3y)e^{y-x^2}$

$critical\ point: \left(-\frac{1}{6}, -\frac{17}{18}\right)$

$f_{xx}(x,y) = +(4x^3 + 12x^2y - 6x - 6y)e^{y-x^2}$

$f_{yy}(x,y) = (3y^2 + xy + 6y + x + 3)e^{y-x^2}$

$f_{xy}(x,y) = (y+1)e^{y-x^2} - 2x(3y^2 + xy + 6y + x + 3)e^{y-x^2}$

$\left(-\frac{1}{6}, -\frac{17}{18}\right)\ is\ local\ minimum$

29. $on\ the\ left\ side, f(0,y) = y, f'(0,y) = 1$
$maximum\ is\ f(0,1) = 1, minimun\ is\ f(0,0) = 0$
$on\ the\ right\ side, f(1,y) = y + 1, f'(1,y) = 1$
$maximum\ is\ f(1,1) = 2, minimum\ is\ f(1,0) = 1$
$on\ the\ up\ side, f(x,1) = x + 1, f'(x+1) = 1$
$maximum\ is\ f(1,1) = 2, minimum\ is\ f(1,0) = 1$
$on\ the\ down\ side\ f(0,y) = y, f'(0,y) = 1$
$maximum\ is\ f(0,1) = 1, minimum\ is\ f(0,0) = 0$
$f_x(x,y) = 1\ and\ f_y(x,y) = 1$
$So, the\ ABS\ MAX\ is\ f(1,1) = 2, ABS\ MIN\ is\ f(0,0) = 0$

35. a) $f_x(x,y) = 1 - 2x - y\ f_y(x,y) = 1 - 2y - x, critical\ point\ \left(\frac{1}{3}, \frac{1}{3}\right)$

$f\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{1}{3}$

b) $on\ the\ left\ side, f(0,y) = y - y^2, f'(0,y) = 1 - 2y$
$maximum\ is\ f\left(0, \frac{1}{2}\right) = \frac{1}{4}, minimun\ is\ f(0,2) = -2$
$on\ the\ right\ side, f(2,y) = -y^2 - y - 2, f'(2,y) = -2y - 1$
$maximum\ is\ f(2,0) = -2, minimum\ is\ f(2,2) = -8$
$on\ the\ up\ side, f(x,2) = -x^2 - x - 2, f'(x,2) = -2x - 1$
$maximum\ is\ f(0,2) = -2, minimum\ is\ f(2,2) = -8$
$on\ the\ down\ side\ f(x,0) = x - x^2, f'(x,0) = 1 - 2x$
$maximum\ is\ f\left(\frac{1}{2}, 0\right) = \frac{1}{4}, minimum\ is\ f(2,0) = -2$

$So, ABS\ MAX\ is \frac{1}{4}\ and\ ABS\ MIN\ is -8$