10/1/20. 14.6 The Chain Rule HW

14.6 #1, 3, 5, 7, 15, 17, 23, 27, 29, 31

1) $f(x,y,z) = x^2y^3 + z^4$

a) $\nabla f = \langle 2xy^3,\ 3y^2x^2,\ 4z^3 \rangle$

b) $x = s^2, \quad y = st^2, \quad z = s^2t$

$\frac{dx}{ds} = 2s \quad \frac{dy}{ds} = t^2 \quad \frac{dz}{ds} = 2st$

c) $\frac{df}{ds} = \frac{df}{dx} \cdot \frac{dx}{ds} + \frac{df}{dy} \cdot \frac{dy}{ds} + \frac{df}{dz} \cdot \frac{dz}{ds}$

$= (2xy^2)(2s) + t^2(3y^2x^2) + 4z^3(2st)$

$= 2s^3t^2(2s) \quad t^2(3s^2t^4 \cdot s^4) + 8(s^6t^3)(2st)$

$= 4s^4t^2 + 3s^6t^6 + 16s^7t^4$.

3) $f(x,y,z) = xy + z^2, \quad x = s^2, \quad y = 2rs, \quad z = r^2$

$\nabla f = \langle y, x, 2z \rangle \quad \frac{dx}{ds} = 2s, \ \frac{dx}{dr} = 0$

$\frac{dy}{ds} = 2r, \ \frac{dy}{dr} = 2s$

$\frac{dz}{ds} = 0, \ \frac{dz}{dr} = 2r$

$\frac{df}{dr} = y(0) + x(2s) + 2z(2r) = 2s^3 + 4r^3$

$\frac{df}{ds} = y(2s) + 2r(x) + 0 = 4rs^2 + 2s^2r$.

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5) $g(x,y) = \cos(x-y)$ $\quad x = 3u - 5v$ $\quad \frac{dx}{du} = 3 \quad \frac{dx}{dv} = ...$

$y = -7u + 15v.$

$\nabla g = \langle \sin(x-y), -\sin(x-y) \rangle \quad \frac{dy}{du} = -7 \quad \frac{dy}{dv} = 15$

$\frac{dg}{du} = \sin(x-y)(3) + -\sin(x-y)(-7)$

$= 3\sin(3u-5v) + 7\sin(3u-5v)$

$\frac{dg}{dv} = \sin(x-y)(-5) + 15(\sin(x-y))$

$-5\sin(3u-5v) + 15\sin(3u-5v)$

7) $\frac{\partial F}{dy}$ $\quad F(u,v) = e^{u+v}$ $\quad U = x^2$

$\langle e^{u+v}, e^{u+v} \rangle$ $\quad V = xy$

$\frac{du}{dy} = 0 \quad \frac{dv}{dy} = x$

$\frac{dF}{dy} = 0(e^{u+v}) + xe^{u+v} = \sqrt{u}\ e^{u+v}$

15) $g(x,y) = x^2 - y^2$ $\quad$ ~~x = e^u~~ $\quad \frac{dx}{du} = e^u$

$\nabla g = \langle 2x, -2y \rangle$ $\quad y =$

$x = e^u \cos v \quad \frac{dx}{dv} = -\sin v\, e^u + e^u \cos v$

$y = e^u \sin v =$ $\quad \cos v\, e^u + \sin v\, e^u$

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$= 2x(-\sin v \cdot e^u + e^u \cos v) \; 2y(\cos v\, e^u + e^u \sin v)$

$u=0 \quad v=1$

⑩ $x = 1 \cdot \cos(1)$

$y = 1 \cdot \sin(1)$

$2\cos(1) \cdot (-\sin(1) + \cos(1)) - 2(\sin(1)) \cdot (\cos(1) + \sin(1))$

(17) $36 + 64 = 10^2$

$\frac{8}{10}(20) + \frac{6}{18}(18)$

$= 26.8$ ft/s

(13) $\left(\frac{df}{dx}\right)^2 - \left(\frac{df}{dy}\right)^2 = \frac{df}{ds} \cdot \frac{df}{dt}$

$\frac{df}{ds} = \frac{df}{dx} \cdot \frac{dx}{ds} \qquad \frac{df}{dy} \frac{dy}{ds} = \frac{df}{ds}$

$\frac{df}{dt} \qquad = \frac{df}{dx} - \frac{df}{dy}$

$\left(\frac{df}{dx} + \frac{df}{dy}\right) \cdot \left(\frac{df}{dx} - \frac{df}{dy}\right) = \left(\frac{df}{dx}\right)^2 - \left(\frac{df}{dy}\right)^2$

27) $\frac{dz}{dx}$, $x^2y + y^2z + xz^2 = 10.$

$$2xy + y^2z + (z^2)\left(2z \cdot \frac{dz}{dx}\right) = 0$$

$F_x = 2xy + z^2$

$F_z = y^2 + 2xz$

$$\frac{dz}{dx} = -\frac{2xy + z^2}{y^2 + 2xz}$$

29) $\frac{dz}{dy}$, $e^{xy} + \sin(xz) + y = 0.$

$F_z = x\cos(xz)$

$F_y = xe^{xy} + 1$

$$= -\frac{xe^{xy} + 1}{x\cos(xz)}$$

31) $\frac{dw}{dy}$ $\quad \frac{1}{w^2 + x^2}$, $\frac{1}{w^2 + y^2}$

$(w^2 + x^2)^{-1}$

$F_w = -(w^2 + x^2)^{-2} \cdot 2w + -(w^2 + y^2)^{-2} \cdot 2w$

$F_y = -(w^2 + y^2)^{-2} \cdot 2y$

$$= -\frac{2w(w^2 + x^2)^{-2} + 2w(w^2 + y^2)^{-2}}{2y(w^2 + y^2)^{-2}}$$

10/6/20 | 14.7 Homework

14.7 # 1, 3, 5, 7, 11, 13, 17, 19, 23, 29, 3

1) $f(x,y) = x^2 + y^4 - 4xy$. (a,b) is a critical point.

a) $f_x = 2x - 4y = 0$
$f_y = 4y^3 - 4x = 0$.

$2x = 4y$
$x = 2y$ ✓
$4y^3 - 4(2y) = 0$
$4y^3 - 8y = 0$
$y(y^2 - 2) = 0$
$y = 0, x = 0$, $y = \sqrt{2}, x = 2\sqrt{2}$, $y = -\sqrt{2}, x = -2\sqrt{2}$

b) $f_{xx} = 2$ $f_{yy} = 12y^2$ $f_{xy} = -4$

At (0,0) $2(0) - (16) = -16$, Saddle point
At $(2\sqrt{2}, \sqrt{2})$ $2(24) - 16 = 32$ Local min!
At $(-2\sqrt{2}, -\sqrt{2})$ $= 32$ Local min!

$8 + 4 - 8 = 4 \to$ Abs min

3) $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_x = 2x + y$

$f_y = 32y^3 + x - 6y - 3y^2$ $\quad y = -2x$

$x = -\frac{y}{2}$

$32y^3 - \frac{y}{2} - 6y - 3y^2 = 0$

$y = -1.458, \quad 0, \quad 4.58$

$x = -0.729, \quad 0, \quad 2.29$

) $f(x,y) = y^2x - yx^2 + xy$

) $f_x = y^2 - 2xy + y = y(y - 2x + 1)$ ✓

$f_y = 2yx - x^2 + x = x(2y - x + 1)$ ✓

) If $y = 0$, $\quad x(-x+1) = 0$

$x = 0$ or 1 $\quad -x^2 + x = 0$

$(1,0)$ or $(0,0)$ $\quad x(x-1) = 0$

If $x = 0$ $\quad$ ~~If x = 0~~

$y(y+1) = 0$

$y^2 + y = 0$, y must be 0.

$f_{xx} = -2x \quad f_{xy} = 2y + 1$

$f_{yy} = 2y$

$(1,0)$ $\quad (-2)(1)(0) - (1)^2 = -1$, Saddle Point.

$(0,0)$ $\quad (0)(0) - 1^2 = -1$, Saddle point.

$2y - \left(\frac{1+y}{2}\right) + 1 = 0$

7) $f(x,y) = x^2 + y^2 - xy + x$

$f_x = 2x - y + 1$

$f_y = 2y - x + 1.$

$2x = \frac{1+y}{2}$

- Crit Points: $(-1,-1)$.

$F_{xx} = 2$ $\quad f_{yy} = 2$

$f_{xy} = -1$

$2(2) - 1 = 3$

$(-1,-1)$ is a local minimum.

11) $f(x,y) = 4x - 3x^3 - 2xy^2$

$f_x = 4 - 9x^2 - 2y^2$ $\quad f_{xx} = -18x$

$f_y = -4xy$ $\quad f_{xy} = -4y$

If $f_y = 0$, $x = 0$ or $y = 0$. $\quad f_{yy} = -4x$

If $x = 0$, $y = \sqrt{2}, -\sqrt{2}$

If $y = 0$, $x = \frac{2}{3}$ or $-\frac{2}{3}$

	D	
$(0, \sqrt{2})$	-32	Saddle
$(0, -\sqrt{2})$	-32	Saddle.
$(-\frac{2}{3}, 0)$	32	Local Max
$(\frac{2}{3}, 0)$	32	Local min

13). $f(x,y) = x^4 + y^4 - 4xy$

$f_x = 4x^3 - 4y$ $\quad y(x^3 - y)$ $\quad f_{xx} = 12x^2$ $\quad f_{xy} = -4$

$f_y = 4y^3 - 4x$ $\quad \boxed{D}$ $\quad f_{yy} = 12y^2$

$(0,0)$ $\quad -16$ $\quad$ Saddle.

$(1,1)$ $\quad 12(12) - 16 > 0.$ $\quad f_{xx}(1,1) > 0$ Local Min

17) $f(x,y) = \sin(x+y) - \cos x$

$f_x = \cos(x+y) + \sin x$

$f_y = \cos(x+y)$ $\quad$ cos 0 at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

$x + y = \frac{(2k+1)\pi}{2}$, $y = \frac{(2k+1)\pi}{2} - x$

$x = k\pi$

$y = \frac{(2n+1)\pi}{2}$

$f_{xx} = -\sin(x+y) + \cos x$

$f_{xy} = -\sin(x+y)$

$f_{yy} = -\sin(x+y)$

$\left(k\pi, \frac{(4n+3)\pi}{2}\right)$

19) $f(x,y) = \ln x + 2\ln y - x - 4y$

$f_x = \frac{1}{x} - 1$ → when $x = 1$, $f_x = 0$.

$f_y = \frac{2}{y} - 4$ → $y = 0.5$, $f_y = 0$

$f_x = 0,\ f_y = 0$

$f_{xx} = -\frac{1}{x^2}$, $f_{yy} = -\frac{2}{y^2}$, $f_{xy} = 0$

P	D	
(1, 0.5)	8	$f_{xx} < 0$, thus it's a local max.

23) $f(x,y) = (x + 3y)e^{y - x^2}$

$f_x = (1 - 2x^2 - 6xy)e^{y-x^2}$

$F_y = (3 + x + 3y)e^{y-x^2}$

$$\left.\begin{array}{l} 1 - 2x^2 - 6xy = 0 \\ 3 + x + 3y = 0. \end{array}\right\} \left(-\frac{1}{6}, -\frac{17}{18}\right).$$

Point	Discrim	
$-\frac{1}{6}, -\frac{17}{18}$	2.57	Local min, $f_{xx} > 0$.

$f_{xx} = (2x^3 + 6x^2y - 3x - 9y)2e^{y-x^2}$

$f_{xy} = (1 - 6xy - 2x^2 - 6x)e^{y-x^2}$

$f_{yy} = (6 + x + 3y)e^{y-x^2}$

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34) $f(x,y) = x+y, \quad 0 \le x \le 1, \quad 0 \le y \le 1$

Max at $x+y = 1+1 = 2$

Min at $x+y = 0+0 = 0$

35) $f_{x,y} = x+y-x^2-y^2-xy$

$f_x = 1-2x-y$
$f_y = 1-2y-x$
$\left.\right\} \left(\frac{1}{3}, \frac{1}{3}\right)$ $y = 1-2x$

a) $f\left(\frac{1}{3}, \frac{1}{3}\right) = \frac{1}{3}$

b) $f(x,0) = x - x^2 = x(1-x)$

$0, 1.$

c)

d) largest value: $\frac{1}{3}$