14.6

1. $f(x,y,z)=x^2y^3+z^4,\ x=s^2,\ y=st^2,\ z=s^2t$

(a). $\frac{df}{dx}=2xy^3,\quad \frac{df}{dy}=3x^2y^2,\quad \frac{df}{dz}=4z^3$

(b). $\frac{dx}{ds}=2s,\quad \frac{dy}{ds}=t^2,\quad \frac{dz}{ds}=2st$

(c).

$$\frac{df}{ds}=\frac{df}{dx}\cdot\frac{dx}{ds}+\frac{df}{dy}\cdot\frac{dy}{ds}+\frac{df}{dz}\cdot\frac{dz}{ds}$$

$$=2xy^3\cdot 2s+3x^2y^2\cdot t^2+4z^3\cdot 2st$$

$$=4xy^3\cdot s+3x^2y^2t^2+8z^3st$$

$$=7s^6t^6+8s^7t^4$$

3. $f(x,y,z)=xy+z^2,\ x=s^2,\ y=2rs,\ z=r^2$

$\frac{df}{dx}=y,\quad \frac{df}{dy}=x,\quad \frac{df}{dz}=2z$

$\frac{dx}{ds}=2s,\quad \frac{dy}{ds}=2r,\quad \frac{dz}{ds}=0$

$\frac{dx}{dr}=0,\quad \frac{dy}{dr}=2s,\quad \frac{dz}{dr}=2r$

$$\frac{df}{ds}=2ys+\cancel{4}2xr$$

$$=6rs^2$$

$$\frac{df}{\cancel{dz}dr}=2xs+4zr$$

$$=2s^3+4r^3$$

5. $g(x,y)=\cos(x-y),\ x=3u-5v,\ y=-7u+15v$

$\frac{dg}{dx}=-\sin(x-y),\quad \frac{dg}{dy}=-\sin(y-x)$

$\frac{dx}{du}=3,\quad \frac{dy}{du}=-7,\quad \frac{dx}{dv}=-5,\quad \frac{dy}{dv}=15$

$$\frac{dg}{du}=-3\sin(x-y)+7\sin(y-x)$$

$$=-10\sin(10u-20v)$$

$$\frac{dg}{dv}=5\sin(x-y)\cancel{+}-15\sin(y-x)$$

$$=20\sin(10u-20v)$$

7. $F(u,v)=e^{u+v}$, $u=x^2$, $v=xy$

$\frac{dF}{du}=e^{\cancel{x+y}\;u+v}$, $\frac{dF}{dv}=e^{u+v}$. $\frac{du}{dx}=2x$, $\frac{dv}{dx}=y$, $\frac{du}{dy}=0$, $\frac{dv}{dy}=x$

$\frac{dF}{dy}=xe^{u+v}=xe^{x^2+xy}$

15. $g(x,y)=x^2-y^2$, $x=e^u\cos v$, $y=e^u\sin v$

$\frac{dg}{dx}=2x-y^2$, $\frac{dg}{dy}=x^2-2y$, $\frac{dx}{du}=e^u\cos v$, $\frac{dy}{du}=e^u\sin v$

$\frac{dg}{du}=(2x-y^2)e^u\cos v+(x^2-2y)e^u\sin v$

$\frac{dg}{du}\Big|_{(0,1)}=2\cos 2$

17.

23. $x=s+t$, $y=s-t$

$\cancel{\frac{df}{dx}}$ $\frac{dx}{ds}=t+1$, $\frac{dx}{dt}=s+1$, $\frac{dy}{ds}=1-t$, $\frac{dy}{dt}=s-1$

27. $x^2y+y^2z+xz^2=10$

$\cancel{\frac{dz}{dx}}$ $2xy+\frac{dz}{dx}y^2+z^2+\frac{dz}{dx}2xz$

$\frac{dz}{dx}=-\frac{2xy+z^2}{2xz+y^2}$

29. $e^{xy}+\sin(xz)+y=0$

$\cancel{ye^{xy}}$ $xe^{xy}+x\cos(xz)\frac{dz}{dy}+1$

$\frac{dz}{dy}=-\frac{xe^{xy}}{x\cos(xz)}$

31. $\frac{1}{w^2+x^2}+\frac{1}{w^2+y^2}=1$

$\frac{dw}{dy}=\frac{-y(w^2+x^2)^2}{w((w^2+x^2)^2+(w^2+y^2)^2)}$

at (1, 1, 1)

$\frac{dw}{dy}=-\frac{1}{2}$

14.7

1. $f(x,y) = x^2 + y^4 - 4xy$

(a) $f_x(x,y) = 2x \cancel{+ y^4} - 4y$

$f_x(a,b) = \cancel{2b + b^4}\ 2a - 4b$

$2a \cancel{2b} - 4b = 0$

$a = 2b$

$f_y(\cancel{a,b}\ x,y) = 4y^3 - 4x$

$f_y(a,b) = \cancel{4}0$

$y^3 = x$

$P = (0,0), (2\sqrt{2}, \sqrt{2})$ or $(-2\sqrt{2}, -\sqrt{2})$

(b). $f_{xx}(x,y) = 2 - 4y$, $f_{xy}(x,y) = 2x - 4$, $f_{yy}(x,y) = 12y^2$

$D(0,0) = -16 < 0$ saddle point

$D(2\sqrt{2}, \sqrt{2}) > 0$, $f_{xx}(2\sqrt{2}, \sqrt{2}) > 0$ local mini

$D(-2\sqrt{2}, -\sqrt{2}) > 0$, $f_{xx}(-2\sqrt{2}, \sqrt{2}) > 0$ local mini

3. $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_x(x,y) = 2x + y$

$f_y(x,y) = 32y^3 + \cancel{y}\,x - 6y - 3y^2$

$f_{xx}(x,y) = 2 + y$, $f_{xy}(x,y) = 2x + 1$, $f_{yy}(x,y) = 96y^2 + x - 6 - 6y$

$2x + y = 0$, $32y^3 + x - 6y - 3y^2 = 0$

$y = -2x$

$x = 0, \frac{13}{64}$, or $-\frac{1}{4} \Rightarrow y = 0, \frac{-13}{32}$, or $\frac{1}{2}$

$(0,0)$ is saddle point, $(\frac{13}{64}, \frac{-13}{32})$ and $(-\frac{1}{4}, \frac{1}{2})$ are local mini

5. $f(x,y) = y^2x - yx^2 + xy$

$f_x(x,y) = y^2 - 2yx + y$, $f_y(x,y) = 2yx - x^2 + x$

$f_x(x,y) = 0 \Rightarrow y(y - 2x + 1) = 0$, $f_y(x,y) = 0 \Rightarrow x(2y - x + 1) = 0$

$\cancel{x=0}$ $(0,0)$, $(1,0)$, $(0,1)$ and $(\frac{1}{3}, -\frac{1}{3})$

$D(\frac{1}{3}, -\frac{1}{3}) > 0$ $f_{xx}(\frac{1}{3}, -\frac{1}{3}) > 0$

$(\frac{1}{3}, -\frac{1}{3})$ is local mini

$(0,0)$, $(1,0)$, and $(0,1)$ are saddle point.

7. $f(x,y)=x^2+y^2-xy+x$

$f_x(x,y)=2x-y+1$

$f_y(x,y)=2y-x$

$f_{xx}(x,y)=2$, $f_{xy}(x,y)=-1$, $f_{yy}(x,y)=2$

$2x-y+1=0$, $2y-x=0$

$x=-\frac{2}{3}$, $y=(-\frac{1}{3})$

$D=4-(-1)^2=3>0$ $f_{xx}>0$ $(-\frac{2}{3},-\frac{1}{3})$ is local mini

11. $f(x,y)=4x-3x^3-2xy^2$

$f_x(x,y)=4-9x^2-2y^2$

$f_y(x,y)=4xy$

$f_{xx}(x,y)=-18x$, $f_{xy}=-4y$ $f_{yy}(x,y)=4x$

$4-9x^2-2y^2=0$, $4xy=0$

$x=0,\frac{2}{3}$ or $-\frac{2}{3}$

when $x=0$, $y=\pm\sqrt{2}$, $x=\frac{2}{3}$, $y=0$, $x=-\frac{2}{3}$, $y=0$

$D_{(0,\pm\sqrt{2})}<0$ $(0,\pm\sqrt{2})$ is saddle point

$D(\frac{2}{3},0)>0$ $f_{xx}(\frac{2}{3},0)<0$ $(\frac{2}{3},0)$ is max point

$D(-\frac{2}{3},0)>0$ $f_{xx}(-\frac{2}{3},0)>0$ $(-\frac{2}{3},0)$ is min point

13. $f(x,y)=x^4+y^4-4xy$

$f_x(x,y)=4x^3+y^4-4y$, $f_y(x,y)=x^4+4y^3-4x$

$f_{xx}(x,y)=12x^2$, $f_{xy}(x,y)=4y^3-4$, $f_{yy}(x,y)=12y^2$

$4x^3+y^4-4y=0$, $x^4+4y^3-4x=0$

$x=0,1,$ or -1 $\Rightarrow y=0,1,$ or -1

$D(0,0)<0$ is saddle point

$D(1,1)>0$ $f_{xx}(1,1)>0$ local mini

$D(-1,-1)>0$ $f_{xx}(-1,-1)>0$ local mini

19. $f(x,y) = \ln x + 2\ln y - x - 4y$

$f_x(x,y) = \frac{1}{x} - 1$, $f_y(x,y) = \frac{2}{y} - 4$

$f_{xx}(x,y) = -\frac{1}{x^2}$, $f_{xy}(x,y) = 0$, $f_{yy}(x,y) = -\frac{2}{y^2}$

$\frac{1}{x} - 1 = 0$, $\frac{2}{y} - 4 = 0$

$x = 1$, $y = \frac{1}{2}$

$D(1, \frac{1}{2}) > 0$ $f_{xx}(1, \frac{1}{2}) < 0$ local maxi

21. $f(x,y) = x - y^2 - \ln(x+y)$

$f_x = 1 - \frac{1}{x+y}$, $f_y = -2y - \frac{1}{x+y}$

$f_{xx} = \frac{1}{(x+y)^2}$, $f_{xy} = \frac{1}{(x+y)^2}$ $f_{yy} = -2 + \frac{1}{(x+y)^2}$

$1 - \frac{1}{x+y} = 0$, $-2y - \frac{1}{x+y} = 0$

$x = \frac{3}{2}$, $y = -\frac{1}{2}$

$D(\frac{3}{2}, -\frac{1}{2}) < 0$ is saddle point

23. $f(x,y) = (x+3y)e^{y-x^2}$

$f_x(x,y) = -(2x^2+6xy-1)e^{y-x^2}$, $f_y(x,y) = (3y+x+3)e^{y-x^2}$

$f_{xx}(x,y) = 2(2x^3+6yx^2-3x-3y)e^{y-x^2}$, $f_{xy}(x,y) = -e^{-x^2}(6xy+2x^2+6x-1)e^{y}$

$f_{yy}(x,y) = (3y+x+6)e^{y-x^2}$

$-(2x^2+6xy-1)e^{y-x^2} = 0$, $(3y+x+3)e^{y-x^2} = 0$

$x = -\frac{1}{6}$, $y = -\frac{17}{18}$

$D(-\frac{1}{6}, -\frac{17}{18}) > 0$, $f_{xx} > 0$ is local mini

35. $f(x,y) = x + y - x^2 - y^2 - xy$

$f_x(x,y) = 1 - 2x - y$, $f_y(x,y) = 1 - 2y - x$

$f_{xx}(x,y) = -2$, $f_{xy}(x,y) = -1$, $f_{yy}(x,y) = -2$

$1 - 2x - y = 0$, $1 - 2y - x = 0$

$x =$