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Calc 3
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14.6 : 1, 3, 5, 7, 15, 17, 23, 27, 29, 31.

#1. Let $f(x,y,z) = x^2y^3 + z^4$ and $x = s^2$, $y = st^2$ and $z = s^2t$.

a. $\frac{\partial f}{\partial x} = 2y^3x$ $\quad \frac{\partial f}{\partial y} = 3x^2y^2$ $\quad \frac{\partial f}{\partial z} = 4z^3$

b. $\frac{\partial x}{\partial s} = 2s$ $\quad \frac{\partial y}{\partial s} = t^2$ $\quad \frac{\partial z}{\partial s} = 2st$

c. $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s} = 2y^3x \cdot 2s + 3x^2y^2 \cdot t^2 + 4z^2 \cdot 2st$

$\frac{\partial f}{\partial s} = 4xsy^3 + 3y^2x^2t^2 + 8stz^2$

#3 $\frac{\partial f}{\partial s}, \frac{\partial f}{\partial r}$; $f(x,y,z) = xy + z^2$
$x = s^2$, $y = 2rs$, $z = r^2$

$\frac{\partial f}{\partial x} = y$ $\quad \frac{\partial f}{\partial y} = x$ $\quad \frac{\partial f}{\partial z} = 2z$

$\frac{\partial x}{\partial s} = 2s$ $\quad \frac{\partial y}{\partial s} = 2r$ $\quad \frac{\partial z}{\partial s} = 0$

$\frac{\partial x}{\partial r} = 0$ $\quad \frac{\partial y}{\partial r} = 2s$ $\quad \frac{\partial z}{\partial r} = 2r$

$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$

$\frac{\partial f}{\partial s} = y \cdot 2s + x \cdot 2r + 2z \cdot 0$

$\frac{\partial f}{\partial s} = 2ys + 2xr \rightarrow 2(2rs)s + 2(s^2)r$

$\boxed{\frac{\partial f}{\partial s} = 6rs^2}$

$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$

$\frac{\partial f}{\partial r} = y \cdot 0 + x \cdot 2s + 2z \cdot 2r$

$\frac{\partial f}{\partial r} = 2sx + 4zr \rightarrow 2s(s^2) + 4r(r^2)$

$\boxed{\frac{\partial f}{\partial r} = 2s^3 + 4r^3}$

#5 $\frac{\partial g}{\partial u}, \frac{\partial g}{\partial v}$ $\quad g(x,y) = \cos(x-y)$
$x = 3u - 5v$ $\quad y = -7u + 15v$

$\frac{\partial g}{\partial x} = -\sin(x-y)$ $\quad \frac{\partial g}{\partial y} = -\sin(x-y)$

$\frac{\partial x}{\partial u} = 3$ $\quad \frac{\partial y}{\partial u} = -7$ $\quad \frac{\partial x}{\partial v} = -5$ $\quad \frac{\partial y}{\partial v} = 15$

$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial u}$

$\frac{\partial g}{\partial u} = -\sin(x-y) \cdot 3 - \sin(x-y) \cdot (-7)$

$\frac{\partial g}{\partial u} = -3(x-y) + 7\sin(x-y)$
$-3((3u-5v)-(-7u+15v)) + 7\sin((3u-5v)-(7u+15v))$

$\boxed{\frac{\partial g}{\partial u} = -3\sin(10u-20v) + 7\sin(10u-20v)}$

$\frac{\partial g}{\partial v} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial v}$

$\frac{\partial g}{\partial v} = -\sin(x-y)\cdot(-5) - \sin(x-y) \cdot 15$

$\frac{\partial g}{\partial v} = 5\sin(x-y) - 15\sin(x-y)$
$5\sin((3u-5v)-(-7u+15v)) - 15\sin((3u-5v)-(-7u+15v))$

$\boxed{\frac{\partial g}{\partial v} = 5\sin(10u-20v) - 15(10u-20v)}$

#7 $\frac{\partial F}{\partial y}$; $F(u,v) = e^{u+v}$, $u = x^2$, $v = xy$

$\frac{\partial F}{\partial u} = e^{u+v}$ $\quad \frac{\partial F}{\partial v} = e^{u+v}$ $\quad \frac{\partial u}{\partial y} = 0$ $\quad \frac{\partial v}{\partial y} = x$

$\frac{\partial F}{\partial y} = \frac{\partial F}{\partial u}\frac{\partial F}{\partial y} + \frac{\partial F}{\partial v}\frac{\partial F}{\partial y} = e^{u+v}(0) + e^{u+v}(x) = xe^{u+v} \longrightarrow xe^{x^2+xy}$

$\boxed{\frac{\partial F}{\partial y} = xe^{x^2+xy}}$

#15 $\frac{\partial g}{\partial u}$ at $(u,v) = (0,1)$, where $g(x,y) = x^2 - y^2$, $x = e^u \cos v$, $y = e^u \sin v$

$\frac{\partial g}{\partial x} = 2x$ $\quad \frac{\partial g}{\partial y} = -2y$ $\quad \frac{\partial x}{\partial u} = \cos v \, e^u$ $\quad \frac{\partial y}{\partial u} = e^u \cos v$

$\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial u} = 2x(\cos v e^u) + (-2y(e^u \cos v)) = 2x\cos v e^u + 2y e^u \cos v$

$\frac{\partial g}{\partial u} = 2(e^u\cos v)(\cos v e^u) + 2(e^u \sin v)(e^u \cos v) = 2e^{2u}(\cos^2 v + \sin v \cos v)$

$\frac{\partial g}{\partial u}(0,1) = 2e^{2(0)}(\cos^2(1) + \sin(1)\cos(1))$

$\boxed{\frac{\partial g}{\partial u} = 2(\cos^2(1) + \sin(1)\cos(1))}$

#23 $x = s+t$ and $y = s-t$ show $\left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2 = \frac{\partial f}{\partial s}\frac{\partial f}{\partial t}$

#27 $\frac{\partial z}{\partial x}$, $x^2y + y^2z + xz^2 = 10$ $\quad \frac{\partial z}{\partial x} = 2xy + y^2\frac{\partial z}{\partial x} + (z^2 + x \cdot 2z\frac{\partial z}{\partial x}) = 0$ $\quad \boxed{\frac{\partial z}{\partial x} = 2xy + y^2\frac{\partial z}{\partial x} + z^2 + 2xz\frac{\partial z}{\partial x}}$

#29 $\frac{\partial z}{\partial y}$ $\quad e^{xy} + \sin(xz) + y = 0$ $\quad \boxed{\frac{\partial z}{\partial y} = ye^{xy} + (-\cos(xz) \cdot (z + x\frac{\partial z}{\partial x})) + 1 = 0}$

#31 $\frac{\partial w}{\partial y}$ $\quad \frac{1}{w^2+x^2} + \frac{1}{w^2+y^2} = 1$ at $(x,y,w) = (1,1,1)$

$(w^2+x^2)^{-1} + (w^2+y^2)' = 1$

$\frac{\partial w}{\partial y} = -1(w^2+x^2) \cdot (2w\frac{\partial w}{\partial y}) - (w^2+x^2) \cdot (2w\frac{\partial w}{\partial y} + 2y) = 0$

$\frac{\partial w}{\partial y}(1,1,1) = -(w^2+1) \cdot (2w\frac{\partial w}{\partial y}) - (w^2+1)(2w\frac{\partial w}{\partial y} + 2) = 0$

$\frac{\partial w}{\partial y}(1,1,1) = -2w^3\frac{\partial w}{\partial y} - 2w\frac{\partial w}{\partial y} - \left(2w^3\frac{\partial w}{\partial y} + 2w^2 + 2w\frac{\partial w}{\partial y} + 2\right)$

$\boxed{\frac{\partial w}{\partial y}(1,1,1) = -2w^3\frac{\partial w}{\partial y} - 2w\frac{\partial w}{\partial y} - 2w^3\frac{\partial w}{\partial y} - 2w^2 - 2w\frac{\partial w}{\partial y} - 2}$