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14,6: 1, 3, 5, 7, 15, 23, 27, 29, 31

1. $f(x, y, z) = x^2 y^3 + z^4$

a. $\frac{\partial f}{\partial x} f(x, y, z) = 2xy^3$

$\frac{\partial f}{\partial y} f(x, y, z) = 3y^2 x^2$

$\frac{\partial f}{\partial z} f(x, y, z) = 4z^3$

b. $\frac{\partial x}{\partial s} x = 2s$ $\frac{\partial y}{\partial s} y = t^2$ $\frac{\partial z}{\partial s} z = 2st$

c. $(2s)^6 + 6(2s)^5 t^2 + 8s^7 + 4$

$\frac{\partial f}{\partial s} = 7s^6 + 6y^3 + 8s^7 + 4$

$$f(x, y, z) = xyz^2, 2rs^3 + r^4$$

$$\frac{\partial f}{\partial s} = 6rs^2$$

$$\frac{\partial f}{\partial r} = 2s^3 + 4r^3$$

$$3. \quad g(x, y) = \cos(x - y)$$

$$= \cos(3u - 5v + 7u - 15v)$$

$$= \cos(10u - 20v)$$

$$\frac{\partial g}{\partial u} g(x, y) = -10 \sin(10u - 20v)$$

$$\frac{\partial g}{\partial v} g(x, y) = 20 \sin(10u - 20v)$$

$$7. \quad F(u, v) = e^{u^2 v}$$

$$u = x^2$$

$$v = xy$$

$$= e^{x^2 + xy}$$

$$\frac{dF}{dy} e^{x^2 + xy} = x e^{x^2 + xy}$$

15

$$e^{2u} \cos^2 v - e^{2u} \sin^2 v$$

22

$$2e^{2u} \cos^2 v - 2e^{2u} \sin^2 v$$

x4

$$2 \cos^2 v - 2 \sin^2 v = 2 \cos 2v$$

23

$$f(x, y) = 2xy$$

$$\delta x = 2y$$

$$(2(s+t))^2 - (2(s-t))^2$$

$$(s+t)^2 4 - (s-t)^2 4 = \checkmark$$

$$\frac{\partial f}{\partial s} = 1$$

$$\frac{\partial f}{\partial t} = 2$$

28

$$\frac{2xy + z^2}{y^2 + 2zx}$$

$$F_z = y^2 + 2zx$$

$$F_x = 2xy + z^2$$

29

$$\frac{x e^{xy} + 1}{x \cos(xz)}$$

$$F_z = x \cos(xz)$$

$$F_y = x e^{xy} + 1$$

$$(w^2 + x^2)^{-1} + (w^2 + y^2)^{-1} = 1$$

$$F_w = - (w^2 + x^2)^{-2} (2w) - (w^2 + y^2)^{-2} (2w)$$

31

$$\frac{y(w^2 + x^2)}{w((w^2 + y^2)^2 + (w^2 + x^2)^2)}$$

$$-\frac{2w}{(w^2 + x^2)^2} - \frac{2w}{(w^2 + y^2)^2}$$

$$y = (w^2 + x^2)^{-1} + (w^2 + y^2)^{-1} = 1$$

$$-\frac{2w(w^2 + y^2)^2 - 2w(w^2 + x^2)^2}{(w^2 + x^2)^2 (w^2 + y^2)^2}$$

$$-\frac{2y}{w^2 + y^2} \cdot \frac{(w^2 + x^2)^2 (w^2 + y^2)}{2(w^2 + y^2)^2 (w^2 + x^2)}$$

14.7: 1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35

1. a. $f(0,0) = 0 + 0 + 4(0)(0) = 0$

$f(2\sqrt{2}, 2\sqrt{2}) = 8 + 4 - 4 = 4$ same for $(-2\sqrt{2}, -2\sqrt{2})$

b. Saddle point, local minimum
 $(0,0)$ $(2\sqrt{2}, 2\sqrt{2})$
 $(-2\sqrt{2}, -2\sqrt{2})$



3. $(0,0)$ saddle point

$(\frac{13}{64}, -\frac{13}{32})$ and $(-\frac{1}{4}, \frac{1}{2})$ local minima

8.

a. $y(y-2x+1) = 0 \Rightarrow y = 0$ $f(0,0) = 0^2 - 0 + 0 = 0$

$x(2y-x+1) = 0 \Rightarrow y = 0$ $f(0,0) = 0 - 0 + 0 + 0 = 0$

b. $y' = 1 \Rightarrow y = 2x + 1$ $x = 0$

$y' = -2 \Rightarrow y = x - 1$ $\frac{1}{2}$

c. $y'' = -2 \Rightarrow 0$

$x'' > 0$

$0, 1, 0$

Saddle point: $(0,0)$, $(1,0)$, $(0,1)$
 Local mini: $(\frac{1}{3}, \frac{1}{3})$

$$f'_x(x,y) = 2x - x + 1$$

$$= x + 1 \quad x = -1$$

$$f'_y(x,y) = 2y - y \quad y = 0$$

$$f_{xx}(x,y) = 1 > 0$$

Local min: $(-1, 0)$

$$f_{yy} = -1 < 0$$

Local max: $(0, 0)$

$$f_x(x,y) = 4 - 9x^2 - y^2$$

$$f_y(x,y) = y^2 \quad y = 0$$

Saddle point

$$(0, \pm\sqrt{2})$$

$$f_{xx}(x,y) = -18x$$

Local min: $(\frac{2}{3}, 0)$

$$f_{yy}(x,y) = 1$$

Local max: $(-\frac{2}{3}, 0)$

$$19. \quad f_x(x,y) = \frac{1}{x} - 4$$

$$\frac{1}{x} = 4$$

$$f_{xx}(x,y) = -\frac{1}{x^2}$$

$$x = 1$$

local max
 $(1, \frac{1}{2})$

$$f_x(x,y) = -4$$

$$f_y(x,y) = 0$$

$$23. \quad f_x(x,y) = e^{y-x^2} - 2x(x+3x)e^{y-x^2}$$

$$f_{xx}(x,y) = -2x e^{y-x^2} (-2x)(x+3x) e^{y-x^2} + (1-2y)(-2x) e^{y-x^2}$$

$$f_y(x,y) = 3e^{y-x^2} + (1-x^2)e^{y-x^2} \quad (x+3x) x = \frac{1}{6}$$

$$f_{yy}(x,y) = 3e^{y-x^2}$$

local min
 $(-\frac{1}{6}, \frac{17}{18})$

29.

$$f_x(x,y) = 1$$

$$f_{xx}(x,y) = 0$$

$$f_y(x,y) = 1$$

$$f_{yy}(x,y) = 0$$

Global max: 2

Global min: 0

33.

$$a) f_x(x,y) = 1 - 2x - x = 1 - 3x$$

$$x = \frac{1}{3}$$

$$f_y(x,y) = 1 - 2x - y = 1 - 3y \quad y = \frac{1}{3}$$

$$b) \begin{matrix} x(x-1) \\ x=0 \\ x=1 \end{matrix} \quad \begin{matrix} \frac{2}{3} - \frac{1}{9} - \frac{1}{9} - \frac{1}{9} \\ \frac{4}{9} - \frac{3}{9} = \frac{1}{9} \end{matrix}$$

extreme value: $\frac{1}{9}$

c)

extreme value: $\frac{1}{3}$

d)

$$\frac{1}{3} \text{ - extreme max}$$

21.

$$P_{xy}(x, y) = 1 - \frac{1}{x+y}$$

$$P_y(x, y) = -2x - \frac{1}{x+y}$$

There is none.

$$\frac{-1}{x+y} = 2x$$

$$2x(x+y) = -1$$

$$2xy + 2y^2 = -1$$