Homework due 10/11

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Sec. 14.6

1) $\frac{df}{dx} = 2x \cdot y^3$ $\frac{df}{dy} = 3x^2 \cdot y^2$ $\frac{df}{dz} = 4z^3$

b) $\frac{dx}{ds} = 2s$ $\frac{dy}{ds} = t^2$ $\frac{dz}{ds} = 2st$

c) $\frac{df}{ds} = \left(2(s^2) \cdot (st^2)^3\right)(2s) + \left(3(s^2)^2 \cdot (st^2)^2\right)(t^2)$
$+ \left(4(s^2t)^3\right)(2st)$

$\frac{df}{ds} = 4s^6t^6 + 3s^6t^6 + 8s^7t^4 = 7s^6t^6 + 8s^7t^4$

3) $\frac{df}{dx} = y$ $\frac{df}{dy} = x$ $\frac{df}{dz} = 2z$ $\frac{dx}{ds} = 2s$ $\frac{dx}{dr} = 0$

$\frac{dy}{ds} = 2r$ $\frac{dy}{dr} = 2s$ $\frac{dz}{ds} = 0$ $\frac{dz}{dr} = 2r$

$$\frac{df}{ds} = (2rs)\cdot(2s) + (s^2)\cdot(2r) + (2r^2)\cdot(0)$$
$$= 4rs^2 + 2rs^2 = 6rs^2$$

$$\frac{df}{dr} = (2rs)\cdot(0) + (s^2)\cdot(2s) + (2r^2)\cdot(2r)$$
$$= 2s^3 + 4r^3$$

5) $\frac{dg}{dx} = -\sin(x-y)\cdot 1 \qquad \frac{dg}{dy} = -\sin(x-y)\cdot(-1)$

$\frac{dx}{du} = 3 \qquad \frac{dx}{dv} = -5 \qquad \frac{dy}{du} = -7 \qquad \frac{dy}{dv} = 15$

$$\frac{dg}{du} = -\sin(10u-20v)\cdot 3 + \sin(10u-20v)\cdot(-7)$$
$$= -10\sin(10u-20v)$$

$$\frac{dg}{dv} = -\sin(10u-20v)\cdot(-5) + \sin(10u-20v)\cdot 15$$
$$= 20\sin(10u-20v)$$

7) $\frac{dF}{du} = e^{u+v}\cdot 1 \qquad \frac{dF}{dv} = e^{u+v}\cdot 1$

$\frac{du}{dx} = 2x \qquad \frac{du}{dy} = 0 \qquad \frac{dv}{dx} = y \qquad \frac{dv}{dy} = x$

$$\frac{dF}{dy} = e^{x^2+xy} \cdot 0 + e^{x^2+xy} \cdot x = x \cdot e^{x^2+xy}$$

15) $\frac{dg}{dx} = 2x \quad \frac{dg}{dy} = -2x \quad \frac{dx}{du} = e^u \cos(v) \quad \frac{dy}{du} = e^u \sin(v)$

$$\frac{dg}{du} = 2\cos(1) \cdot \cos(1) - 2\sin(1) \cdot \sin(1) = 2\cos(2)$$

17) $\frac{-18x}{\sqrt{x^2+y^2}} + \frac{-20x}{\sqrt{x^2+y^2}}$

$$\frac{-18 \cdot 6}{\sqrt{6^2+8^2}} + \frac{-20 \cdot 8}{\sqrt{6^2+8^2}} = -26.8$$

23) $\frac{df}{ds} = \frac{df}{dx} \cdot \frac{dx}{ds} + \frac{df}{dy} \cdot \frac{dy}{ds} \qquad \frac{dx}{ds} = 1 \quad \frac{dy}{ds} = 1$

$$\frac{df}{dt} = \frac{df}{dx} \cdot \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt} \qquad \frac{dx}{dt} = 1 \quad \frac{dy}{dt} = -1$$

$$\frac{df}{ds} \cdot \frac{df}{dt} = \left(\frac{df}{dx} + \frac{df}{dy}\right) \cdot \left(\frac{df}{dx} - \frac{df}{dy}\right) = \left(\frac{df}{dx}\right)^2 - \left(\frac{df}{dy}\right)^2$$

27) $2xy + y^2 \frac{dz}{dx} + z^2 + 2xz\frac{dz}{dx} = 0$

$$\frac{dz}{dx}\left(y^2+2xz\right) = -2xy - z^2$$

$$\frac{dz}{dx} = -\frac{2xy+z^2}{y^2+2xz}$$

29) $e^{xy}\cdot x + \cos(xz)\cdot x\,\frac{dz}{dy} + 1 = 0$

$$\left(\cos(xz)\cdot x\right)\frac{dz}{dy} = -xe^{xy} - 1$$

$$\frac{dz}{dy} = -\frac{xe^{xy}+1}{x\cos(xz)}$$

31) $-\left(w^2+x^2\right)^{-2}\cdot 2w\,\frac{dw}{dy} - \left(w^2+y^2\right)^{-2}\cdot\left(2w\frac{dw}{dy}+2y\right)$

$$-\frac{1}{2^{\cancel{2}}}\cdot\cancel{2}\frac{dw}{dy} - \frac{1}{2^{\cancel{2}}}\cdot\cancel{2}\left(\frac{dw}{dy}+1\right) = 0$$

$$-\frac{1}{2}\frac{dw}{dy} - \frac{1}{2}\frac{dw}{dy} - \frac{1}{2} = 0$$

$$\frac{dw}{dy} = -\frac{1}{2}$$

Sec. 14.7

1) $f_x(x,y) = 2x - 4y$

$f_x(a,b) = 2a - 4b$

$2a - 4b = 0$

$a = 2b$

$f_y(x,y) = 4y^3 - 4x$

$4b^3 - 4a = 0$

$f_y(a,b) = 4b^3 - 4a$

$b^3 - 2b = 0$

$b(b^2 - 2) = 0$

$a = 0, 2\sqrt{2}, -2\sqrt{2}$ $\quad$ $b = 0, \sqrt{2}, -\sqrt{2}$

b) $f_{xx} = 2$ $\quad$ $f_{yy} = 12y^2$ $\quad$ $f_{xy} = -4$

$D(0,0) = -16$ $\quad$ saddle point

$D(2\sqrt{2}, \sqrt{2}) = 2 \cdot 24 - 16 = 32$

$f_{xx}(2\sqrt{2}, \sqrt{2}) = 2$ $\quad$ local min

$D(-2\sqrt{2}, \sqrt{2}) = 32$

$f_{xx}(-2\sqrt{2}, \sqrt{2}) = 2$ $\quad$ Local min

absolute min value is -4

3) $f_x = 2x + y$ $\quad f_y = 32y^3 + x - 6y - 3y^2$

$P = (0,0)$, $(13/64, -13/32)$, $(-1/4, 1/2)$

saddle point $\quad$ minima $\quad$ minima

5) $f_x = y^2 - 2yx + y = y(y - 2x + 1)$ ✓

$f_y = 2yx - x^2 + x = x(2y - x + 1)$ ✓

b) $P = (0,0)$, $(0,-1)$, $(1,0)$, $(1/3, -1/3)$

c) $f_{xx} = -2y$ $\quad f_{yy} = 2x$ $\quad f_{xy} = 2y - 2x + 1$

$D(0,0) = -1$ saddle point

$D(0,-1) = -1$ saddle point

$D(1,0) = -1$ saddle point

$D(1/3, -1/3) = \frac{1}{3}$ $\quad f_{xx}(1/3, -1/3) = \frac{2}{3}$ $\quad$ min

7) $f_x = 2x - y + 1$ $\quad f_y = 2y - x$

$f_{xx} = 2$ $\quad f_{yy} = 2$ $\quad f_{xy} = -1$

$D = 4 - 1 = 3$ $\quad P = (-2/3, -1/3)$

$f_{xx}(-2/3, -1/3) = 2$ $\quad$ MIN

11) $f_x = 4 - 9x^2 - 2y^2$ $f_y = -4xy$

$f_{xx} = -18x$ $f_{yy} = -4x$ $f_{xy} = -4y$

$P = \left(-\frac{2}{3}, 0\right), \left(0, -\sqrt{2}\right), \left(0, \sqrt{2}\right), \left(\frac{2}{3}, 0\right)$

$D = +$ $D = -$ $D = -$ $D = +$

$f_{xx} = 12$ saddle point saddle point $f_{xx} = -12$

MIN MAX

13) $f_x = 4x^3 - 4y$ $f_y = 4y^3 - 4x$

$f_{xx} = 12x^2$ $f_{yy} = 12y^2$ $f_{xy} = -4$

$P = (-1, -1), (0, 0), (1, 1)$

$D = +$ $D = -$ $D = +$

$f_{xx} = +$ saddle point $f_{xx} = +$

MIN MIN

19) $f_x = \frac{1}{x} - 1$ $f_y = \frac{2}{y} - 4$

$f_{xx} = -x^{-2}$ $f_{yy} = -2y^{-2}$ $f_{xy} = 0$

$P = \left(1, \frac{1}{2}\right)$ $D = +$ $f_{xx} = -1$ MAX

23) $f_x = -(2x^2 + 6xy - 1) \cdot e^{y-x^2}$ $f_y = (3y + x + 3) \cdot e^{y-x^2}$

$f_{xx} = (4x^3 + 12x^2y - 6x - 6y) \cdot e^{y-x^2}$ $f_{xy} = (1 - 2x^2 - 6xy - 6x) \cdot e^{y-x^2}$

$f_{yy} = (x + 3y + 6) \cdot e^{y-x^2}$

$P = (-1/6, -12/18)$ $D = -$ Saddle point.

29) Max: $(1,1) = 2$ Min: $(0,0) = 0$

35) $f_x = 1 - 2x - y$ $f_y = 1 - 2y - x$

$P = (1/3, 1/3)$ ✓ in square

$f(1/3, 1/3) = 1/3$

Bottom: $f(x,0) = x - x^2$ $P = (0,0), (1,0)$

$f(0,0) = 0$ $f(1,0) = 0$

Top: $f(x,2) = -x - x^2 - 2$ $P =$ no real solutions

Left: $f(0,y) = y - y^2$ $P = (0,0), (0,1)$

$f(0,0) = 0$ $f(0,1) = 0$

Right: $f(2,y) = -y - y^2 - 2$ $P =$ no real solutions

Abs. Max: $1/3$ at $(1/3, 1/3)$

Abs Min: 0 at $x=0$ and $y=0$