14.6 - # 1, 3, 5, 7, 15, 23, 27, 29, 31

1. $f(x,y,z) = x^2y^3 + z^4$ $\quad x = s^2$, $y = st^2$, $z = s^2t$

$\frac{df}{dx} = 2xy^3$

$\frac{df}{dy} = 3x^2y^2$

$\frac{df}{dz} = 4z^3$

3. $\frac{df}{ds} = \frac{df}{dx}\frac{dx}{ds} + \frac{df}{dy}\frac{dy}{ds} + \frac{df}{dz}\frac{dz}{ds}$

$\frac{dx}{ds} = 2s$

$\frac{dy}{ds} = t^2$

$\frac{dz}{ds} = 2st$

$\frac{df}{ds} = (2xy^3)(2s) + (3x^2y^2)(t^2) + (4z^3)(2st)$

$\frac{df}{ds} = 4xy^3s + 3x^2y^2t^2 + 8z^3st$

5. $g(x,y) = \cos(x-y)$

$x = 3u - 5v$

$y = -7u + 15v$

$\frac{dg}{du} = \frac{dg}{dx}\frac{dx}{du} + \frac{dg}{dy}\frac{dy}{du}$

$= (-\sin(x-y))(3) + (-\sin(x-y)(-1))(-7)$

$= \sin(x-y)(-3-7)$

$= -10\sin(x-y)$

$\frac{dg}{dv} = \frac{dg}{dx}\frac{dx}{dv} + \frac{dg}{dy}\frac{dy}{dv}$

$= -\sin(x-y)(-5) + (-\sin(x-y)(-1))(15)$

$= \sin(x-y)(5+15)$

$= 20\sin(x-y)$

7. $\frac{dF}{dy}$; $F(u,v) = e^{u+v}$, $u = x^2$, $v = xy$

$\frac{dF}{dy} = \frac{dF}{dv}\frac{dv}{dy} + \frac{dF}{du}\frac{du}{dx}$

$= (e^{u+v})(x) + (e^{u+v})(y)$

$= (e^{u+v})(x+y)$

15. $\frac{dg}{du} = ?$ @ $(u,v) = (0,1)$

$g(x,y) = x^2 - y^2$

$x = e^u\cos(v) \Rightarrow x(0) = e^0\cos(0) = 1$

$y = e^u\sin(v) \Rightarrow y(1) = e^1\sin(1) = e$

$\frac{dg}{du} = \frac{dg}{dx}\frac{dx}{du} + \frac{dg}{dy}\frac{dy}{du}$

$= (2x)(e^u\cos v) + (-2y)(e^u\sin(v))$

$\frac{dg}{du}(0,1) = 2(0) - 2e(e\sin(e))$

$= -2e^2\sin(e)$

23. $x = s+t$, $y = s-t$

$\left(\frac{df}{dx}\right)^2 - \left(\frac{df}{dy}\right)^2 = \frac{df}{ds}\frac{df}{dt} = (1+1)(1-1) = 0$

27. $\frac{dz}{dx}$, $x^2y + y^2z + xz^2 - 10 = 0$

$f_x = 2xy + z^2$

$f_y = x^2 + 2yz$

$f_z = y^2 + 2xz$

$\frac{dz}{dx} = \frac{-f_x}{f_z}$

$\frac{dz}{dx} = \frac{-(2xy+z^2)}{(y^2+2xz)}$

30. $\frac{dz}{dy}$, $e^{xy} + \sin(xz) + y = 0$

$f_x = (e^{xy})(y) + \cos(xz)(z)$ $\quad \frac{dz}{dy} = \frac{-f_y}{f_z} = \frac{-(xe^{xy}+1)}{x\cos(xz)}$

$f_y = (e^{xy})(x) + 1$

$f_z = \cos(xz)(x)$

31. $\frac{dw}{dy} = \frac{1}{w^2+x^2} + \frac{1}{w^2+y^2} = 1 \quad @ \quad (x,y,w) = (1,1,1)$

$f_x = -1(w^2+x^2)^{-2}(2x) = \frac{1}{-2x(w^2+x^2)^2}$

$f_y = -1(w^2+y^2)^{-2}(2y) = \frac{1}{-2y(w^2+y^2)^2}$

$f_w = -(w^2+x^2)^{-2}(2w) + -(w^2+y^2)^{-2}(2w) = \frac{-1}{2w(w^2+x^2)^2} - \frac{1}{2w(w^2+y^2)^2}$

$$\frac{dw}{dy} = \frac{-f_y}{f_w} = \left(\frac{1}{-2x(w^2+x^2)^2}\right)\left(\frac{-2w(w^2+x^2)^2}{1} - \frac{2w(w^2+y^2)^2}{1}\right)$$

$$= \frac{2w(w^2+x^2)^2}{2x(w^2+x^2)^2} + \frac{2w(w^2+y^2)^2}{2x(w^2+x^2)^2}$$

$$= \frac{w}{x} + \frac{w(w^2+y^2)^2}{x(w^2+y^2)^2} \quad @ \quad (1,1) = 1 + 1 = 2$$

13. $f(x,y) = x^4 + y^4 - 4xy$

$f_x = 4x^3 - 4y = 0 \rightarrow y = x^3$

$f_y = 4y^3 - 4x = 0$

$4x^3 - 4x = 0$

$4x(x^2 - 1) = 0$

$x = 0, +1, -1$

$y = x^3$

$y(0) = (0)^3 = 0$

$y(-1) = (-1)^3 = -1$

$y(1) = (1)^3 = 1$

$P = (0,0); (-1,-1), (1,1)$

19. $f(x,y) = \ln x + 2\ln y - x - 4y \quad P(1,2)$

$f_x = \frac{1}{x} - 1 = 0 \rightarrow x = 1$

$f_y = \frac{2}{y} - 4 = 0$

$2 = 4y \rightarrow y = 2$

21. $f(x,y) = x - y^2 - \ln(x+y)$

$f_x = 1 - \frac{1}{x+y} = 0 \Rightarrow x + y = 1$

$y = 1 - x$

$f_y = -2y - \frac{1}{x+y} = 0$

$-2(1-x) - \frac{1}{(x + (1-x))} = 0$

$-2 + 2x - 1 = 0$

$x = 3/2$

$1 - \frac{1}{\frac{3}{2} + y} = 0 \rightarrow \frac{3}{2} + y = 1$

$y = -\frac{1}{2}$

$P\left(\frac{3}{2}, -\frac{1}{2}\right)$

23. $f(x,y) = (x+3y)e^{y-x^2}$

$f_x = (1+3y)(e^{y-x^2}) + (x+3y)(e^{y-x^2})(-2x)$

$f_y = (x+3)e^{y-x^2} + (x+3y)(e^{y-x^2})(1)$

29. $f(x,y) = x + y$

$f_x = 1 + y = 0 \rightarrow y = -1$

$f_y = x + 1 = 0 \rightarrow x = -1$

$f(-1,-1) = -1 + (-1) = -2$

no extrema $0 \le x \le 1$ and $0 \le y \le 1$

35. $f(x,y) = x + y - x^2 - y^2 - xy$

$f_x = 1 - 2x - y = 0 \rightarrow y = 1 - 2x$

$f_y = 1 - 2y - x = 0$

$f_y = 1 - 2(1-2x) - x = 0$

$1 - 2 + 4x - x = 0$

$-1 = -3x \rightarrow x = 1/3$

$1 - 2y - 1/3 = 0$

$-2y = -2/3$

$y = -2/-6 = 1/3$

$P(1/3, 1/3)$

$f(1/3, 1/3) = \frac{1}{3} + \frac{1}{3} - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)^2 - \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)$

14.7 - #1, 3, 5, 7, 11, 13, 19, 21, 23, 29, 35

1. $f(x,y) = x^2 + y^4 - 4xy$

$f_x(x,y) = 2x - 4y = 0 \rightarrow 2x - 4y = 4y^3 - 4x$

$f_y(x,y) = 4y^3 - 4x = 0$

$\frac{6x}{6} = \frac{(4y^3 + 4y)}{6} \rightarrow x = \frac{2}{3}y^3 + \frac{2}{3}y$

$2\left(\frac{2}{3}y^3 + \frac{2}{3}y\right) - 4 = 0 \rightarrow \frac{4}{3}y^3 + \frac{4}{3}y = 4$

$\frac{4}{3}y(y^2 + y) = 4$

$y = 0$

$2x - 0 = 0 \rightarrow x = 0$

$\boxed{P = (0,0)}$

3. $f(x,y) = 8y^4 + x^2 + xy - 3y^2 - y^3$

$f_x = 2x + y = 0 \rightarrow y = -2x$

$f_y = 32y^3 + x - 6y - 3y^2$

$f_y = 32(-2x)^3 + x - 6(-2x) - 3(-2x)^2 = 32(-8)x^3 + x + 12x - 12x^2$

$f_y = 256x^3 + 13x - 12x^2 = x(256x^2 - 12x + 13)$

$x = 0 \rightarrow y = 0$

5. $f(x,y) = y^2x - y^2x + xy$

$y(y - 2x + 1) = 0 \quad x(2y - x + 1) = 0$

$f_x = y^2 - y^2 + y = y$

$f_y = 2yx - 2yx + y = y$

7. $f(x,y) = x^2 + y^2 - xy + x$

$f_x = 2x - y + 1 = 0 \rightarrow 2(2y) - y + 1 = 0 \rightarrow 4y - y + 1 \Rightarrow 3y = -1 \rightarrow y = -1/3$

$f_y = 2y - x = 0 \rightarrow x = 2y \rightarrow x = 2(-1/3) = -2/3 \quad P = (-1/3, -2/3)$

11. $f(x,y) = 4x - 3x^3 - 2xy^2$

$f_x = 4 - 9x^2 - 4y = 0 \rightarrow 4 = 4y \rightarrow y = 1 \quad P(0,1)$

$f_y = -4x = 0 \rightarrow x = 0$