14.6 HW

1. $f(x,y,z) = x^2y^3 + z^4 \quad x = s^2 \quad y = st^2 \quad z = s^2t$

a) $\frac{df}{dx} = 2xy^3 \quad \frac{df}{dy} = 3x^2y^2 \quad \frac{df}{dz} = 4z^3$

b) $\frac{dx}{ds} = 2s \quad \frac{dy}{ds} = t^2 \quad \frac{dz}{ds} = 2st$

c) $df/ds = 7s^6t^6 + 8s^7t^4$

3. $df/ds = 6rs^2 \quad df/dr = 2s^3 + 4r^3$

5. $\frac{dg}{du}, \frac{dg}{dv}$; $g(x,y) = \cos(x-y)$, $x = 3u - 5v$, $y = -7u + 15v$

$\frac{dx}{du} = 3 \quad \frac{dy}{du} = -7 \quad \frac{dx}{dv} = -5 \quad \frac{dy}{dv} = 15 \quad \frac{dg}{du} = \frac{dg}{dx}\cdot\frac{dx}{du} + \frac{dg}{dy}\cdot\frac{dy}{du} = -10\sin(10u - 20v)$

$\frac{dg}{dv} = \frac{dg}{dx}\cdot\frac{dx}{dv} + \frac{dg}{dy}\cdot\frac{dy}{dv} = 20\sin(10u - 20v)$

7. $\frac{dF}{dy}$; $F(u,v) = e^{u+v}$, $u = x^2$, $v = xy \quad \frac{dF}{du} = e^{u+v} \quad \frac{dF}{dv} \quad \frac{du}{dy} = 0 \quad \frac{dv}{dy} = x$

$\frac{dF}{dy} = 0 + xe^{u+v} = \boxed{xe^{x(x+y)}}$

15. dg/du at $(u,v) = (0,1)$, where $g(x,y) = x^2 - y^2$, $x = e^u\cos v$ $y = e^u\sin v$

$\frac{dg}{dx} = 2x \quad \frac{dg}{dy} = -2y \quad \frac{dx}{du} = e^u\cos v \quad \frac{dy}{du} = e^u\sin v$

$dg/du = 2xe^u\cos v - 2ye^u\sin v$ at $(u,v) = (0,1)$ $= \boxed{2\cos(2)}$

27. $\frac{dz}{dx}$, $x^2y + y^2w + xz^2 = 10 \quad F_{xyz} = x^2y + y^2w + xz^2 - 10$

$F_x = 2xy + z^2 \quad F_z = y^2 + 2xz \quad \boxed{\frac{dz}{dx} = -\frac{2xy + z^2}{2xz + y^2}}$

29. $\frac{dz}{dy}$, $e^{xy} + \sin(xz) + y = 0 \quad F_y = xe^{xy} + 1 \quad F_x = x\cos(xz) \quad \frac{dz}{dy} = -\frac{F_y}{F_x} = \boxed{\frac{-xe^{xy}+1}{x\cos(xz)}}$

31. $\frac{dw}{dy}$, $\frac{1}{w^2+x^2} + \frac{1}{w^2+y^2} = 1$ at $(x,y,w) = (1,1,1) \quad F_y = -\frac{2y}{(w^2+y^2)^2}$

$F_w = \frac{-2w}{(w^2+x^2)^2} - \frac{2w}{(w^2+y^2)^2}$

$\frac{dw}{dy}(1,1,1) = -\frac{1}{2}$

(a,b) is critical point

14.7 HW

1. $f(x,y) = x^2+y^4-4xy$ $\quad f_x(x,y) = 2x-4y = 0 \quad a=2b$

$f_y = 4y^3-4x = 0 \quad b=0, \pm\sqrt{2}$

$f_{xx}=2 \quad f_{yy}=12y^2 \quad f_{xy}=-4$

minimum value is 4

3. $f(x,y) = 8y^4+x^2+xy-3y^2-y^3$ $\quad f_x = 2x+y=0 \quad f_y = 32y^3+x-6y-3y^2=0$

$y=-2x \quad 256x^3+x+12x-12x^2=0$

$x=0, \; x=\frac{-12\pm 116}{512} \; ; \; x=\frac{13}{64}$ or $-1/4$

$x = 0, -1/4, 13/64$
$y = 0, 1/2, -13/32$

5. $f(x,y) = y^2x-yx^2+xy$ $\quad f_x = y^2-2xy+y=0 \quad f_y = 2yx-x^2+x=0$

$y=2x-1 \quad x=0$ or 1

$y=-1$ or 1

(0,0), (1,1), and (0,-1) are saddle points

7. $f(x,y) = x^2+y^2-xy+x$ $\quad f_x = 2x-y+1=0 \quad 4y-y+1=0 \quad 3y=-1$

$f_y = 2y-x=0 \quad x=2y \quad y=-1/3$

$x=-2/3$

$f_{xx}=2$

$f_{yy}=2$

$f_{xy}=-1$

$(-2/3, -1/3)$ is a local min

11. $f(x,y) = 4x-3x^3-2xy^2$ $\quad f_x = 4-9x^2-2y^2 \quad f_y = -4xy \quad f_y=0 \quad f_x=0$

$y=\pm\sqrt{2}$ points: $(0,\sqrt{2}), (0,-\sqrt{2}), (2/3,0), (-2/3,0)$

$f_{xx}=-18x$

$f_{yy}=-4x \quad (0,\sqrt{2})$ is a saddle pt

$f_{xy}=-4y \quad (0,-\sqrt{2})$ is a saddle pt

$(2/3,0)$ local max, $(-2/3,0)$ local min

13. x^4+y^4-4xy

(1,1) and (-1,-1): local min

(0,0): saddle point

19. $f(x,y) = \ln x + 2\ln y - x - 4y$ $\quad f_x = \frac{1}{x}-1 \quad f_y = \frac{2}{y}-4 \quad f_{xx} = \frac{-1}{x^2}$

$f_{yy} = \frac{-2}{y^2}$

$f_{xy}=0$

$(1, 1/2)$ is a local min

21. $f(x,y) = x - y^2 - \ln(x+y)$

$f_x = 1 - \frac{1}{x+y}$ $\quad f_y = -2y - \frac{1}{x+y}$

$f_{xx} = \frac{1}{(x+y)^2}$ $\quad f_{yy} = -2 + \frac{1}{(x+y)^2}$ $\quad f_{xy} = \frac{1}{(x+y)^2}$

$\boxed{(3/2, -1/2) \text{ is a saddle point}}$

29. $f(x,y) = x+y \quad 0 \le x \le 1, \; 0 \le y \le 1$

max when $f(1,1) = 2$

min when $f(0,0) = 0$