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Dr 2 - Calc 3
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HW 5: 14 6-7

$\boxed{14.6}$ 1) a) $\frac{\partial f}{\partial x} = 2xy^3, \frac{\partial f}{\partial y} = 3x^2y^2, \frac{\partial f}{\partial z} = 4z^3$

b) $\frac{\partial x}{\partial s} = 2s, \frac{\partial y}{\partial s} = t^2, \frac{\partial z}{\partial s} = 2st$

c) $\frac{\partial f}{\partial s} = 2xy^3(2s) + 3x^2y^2(t^2) + 4z^3(2st)$

$= 2(s^2)(st^2)^3(2s) + 3(s^2)^2(st^2)^2(t^2) + 4(s^2t)^3(2st)$

$= 4s^2s^3t^6s + 3s^4s^2t^4t^2 + 8s^6t^3st$

$= 4s^6t^6 + 3s^6t^6 + 8s^7t^4 = 7s^6t^6 + 8s^7t^4$

3) $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial s}$

$\frac{\partial f}{\partial x} = y, \frac{\partial f}{\partial y} = x, \frac{\partial f}{\partial z} = 2z$

$\frac{\partial x}{\partial s} = 2s, \frac{\partial y}{\partial s} = 2r, \frac{\partial z}{\partial s} = 0$

$\frac{\partial f}{\partial s} = 2ys + 2xr = 4s^2r + 2s^2r = 6s^2r$

$\frac{\partial f}{\partial r} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial r} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial r}$

$\frac{\partial x}{\partial r} = 0, \frac{\partial y}{\partial r} = 2s, \frac{\partial z}{\partial r} = 2r$

$\frac{\partial f}{\partial r} = 2xs + 4zr = 2s^3 + 4r^3$

5) $\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial u}$

$\frac{\partial g}{\partial x} = -\sin(x-y), \frac{\partial g}{\partial y} = \sin(x-y)$

$\frac{\partial x}{\partial u} = 3, \frac{\partial y}{\partial u} = -7$

$\frac{\partial g}{\partial u} = -3\sin(x-y) - 7\sin(x-y) = -10\sin(x-y)$

$= -10\sin(3u-5v+7u-15v) = -10\sin(10u-20v)$

$\frac{\partial g}{\partial v} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial v}$

$\frac{\partial x}{\partial v} = -5, \frac{\partial y}{\partial v} = 15$

$\frac{\partial g}{\partial v} = 5\sin(x-y) + 15\sin(x-y) = 20\sin(10u-20v)$

7) $\frac{\partial F}{\partial y} = \frac{\partial F}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial F}{\partial v}\frac{\partial v}{\partial y}$

$\frac{\partial F}{\partial u} = \frac{\partial F}{\partial v} = e^{u+v}, \frac{\partial u}{\partial y} = 0, \frac{\partial v}{\partial y} = x$

$\frac{\partial F}{\partial y} = xe^{u+v} = xe^{x^2+xy}$

15) $\frac{\partial g}{\partial u} = \frac{\partial g}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial g}{\partial y}\frac{\partial y}{\partial u}$

$\frac{\partial g}{\partial x} = 2x, \frac{\partial g}{\partial y} = -2y, \frac{\partial x}{\partial u} = e^u\cos v, \frac{\partial y}{\partial u} = e^u\sin v$

$= 2\cos 1, = -2\sin 1, = \cos 1, = \sin 1$

$= 2\cos^2 1 - 2\sin^2 1$

23) $\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial s}, \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$

$\frac{\partial f}{\partial s} = \frac{\partial f}{\partial x}(1) + \frac{\partial f}{\partial y}(1), \frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}(1) + \frac{\partial f}{\partial y}(-1)$

$\frac{\partial f}{\partial s}\frac{\partial f}{\partial t} = \left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\right)\left(\frac{\partial f}{\partial x} - \frac{\partial f}{\partial y}\right) = \left(\frac{\partial f}{\partial x}\right)^2 - \left(\frac{\partial f}{\partial y}\right)^2$

27) $0 = 2xy + y^2z' + (z^2 + 2xzz')$

$= 2xy + z^2 + z'(y^2 + 2xz)$

$-2xy - z^2 = z'(y^2+2xz) = \frac{dz}{dx} = -\frac{2xy+z^2}{y^2+2xz}$

29) $0 = xe^{xy} + z'\cos xz + 1$

$xz'\cos xz = -xe^{xy} - 1 \Rightarrow \frac{dz}{dx} = -\frac{xe^{xy}+1}{x\cos xz}$

31) $0 = -2ww'(w^2+x^2)^{-2} - (2ww'+2y)(w^2+y^2)^{-2}$

$= -\frac{2ww'}{(w^2+x^2)^2} - \frac{2ww'+2y}{(w^2+y^2)^2}$

$= \frac{-2ww'(w^2+y^2)^2 - (2ww'+2y)(w^2+x^2)^2}{(w^2+x^2)^2(w^2+y^2)^2}$

$= \frac{-2ww'(w^2+y^2)^2 - 2ww'(w^2+x^2)^2 - 2y(w^2+x^2)^2}{(w^2+x^2)^2(w^2+y^2)^2}$

$\frac{2y(w^2+x^2)^2}{(w^2+x^2)^2(w^2+y^2)^2} = w'\left(\frac{-2w((w^2+y^2)^2+(w^2+x^2)^2)}{(w^2+x^2)^2(y^2+w^2)^2}\right)$

$\frac{dw}{dy} = \frac{2y(w^2+x^2)^2}{-2w((w^2+y^2)^2+(w^2+x^2)^2} = \frac{-y(w^2+x^2)^2}{w((w^2+y^2)^2+(w^2+y^2)^2)}$

$\boxed{14.7}$ 1) a) $f_x = 0 = 2x - 4y, 4y = 2x, 2y = x, a = 2b$

$f_y = 0 = 4y^3 - 4x, 4x = 4y^3, x = y^3$

$2b = b^3, 2 = b^2, b = \sqrt{2}, a = 2\sqrt{2}$

$P = (0,0), (2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2})$

b) maxima: none

minima: $(2\sqrt{2}, \sqrt{2}), (-2\sqrt{2}, -\sqrt{2})$

saddle point: $(0,0)$

3) $f_x = 2x + y, f_y = 32y^3 + x - 6y - 3y^2$

$f_{xx} = 2, f_{xy} = 1, f_{yy} = 96y^2 - 6 - 6y$

$y = -2x, 0 = 32(-2x)^3 + x - 6(-2x) - 3(-2x)^2$

$= 32(-8x^3) + x + 12x - 12x^2$

$= -256x^3 - 12x^2 + 13x = -256x^2 - 12x + 13$

$= (-64x+13)(4x+1), x = \frac{13}{64}, -\frac{1}{4}, y = -\frac{13}{32}, \frac{1}{2}$

saddle point: $(0,0)$

maxima: none, minima: $\left(\frac{13}{64}, -\frac{13}{32}\right), \left(-\frac{1}{4}, \frac{1}{2}\right)$

5) a) $f_x = y^2 - 2xy + y = y(y - 2x + 1) = 0$

$f_y = 2xy - x^2 + x = x(2y - x + 1) = 0$

b) $x = 0: y(y+1) = 0, y = -1 \Rightarrow (0,-1)$ and $(0,0)$

$y = 0: x(1-x) = 0, x = 1 \Rightarrow (1,0)$

$y = 2x - 1 \Rightarrow 0 = x(2(2x-1) - x + 1) = x(4x - 2 - x + 1) = x(3x-1), x = \frac{1}{3}$

$y = -\frac{1}{3}$

CRITICAL POINTS: $(0,0), (0,-1), (1,0), \left(\frac{1}{3}, -\frac{1}{3}\right)$

$\Longrightarrow$

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$\boxed{14.7}$ 5) c) $f_{xx} = -2y,\ f_{xy} = 2y - 2x + 1,\ f_{yy} = 2x$

$f_{xx}(0,0) = 0,\ f_{xx}(0,-1) = 2,\ f_{xx}(1,0) = 0,\ f_{xx}(\frac{1}{3},-\frac{1}{3}) = \frac{2}{3}$

$f_{xy}(0,0) = 1,\ f_{xy}(0,-1) = -1,\ f_{xy}(1,0) = -1,\ f_{xy}(\frac{1}{3},-\frac{1}{3}) = -\frac{1}{3}$

$f_{yy}(0,0) = 0,\ f_{yy}(0,-1) = 0,\ f_{yy}(1,0) = 2,\ f_{yy}(\frac{1}{3},-\frac{1}{3}) = \frac{2}{3}$

$D = f_{xx}f_{yy} - [f_{xy}]^2 = (0)(0) - (1) = -1$
$= (2)(0) - (1) = -1$
$= (0)(2) - (1) = -1$ SADDLE POINTS $(D<0)$
$= (\frac{2}{3})(\frac{2}{3}) - (\frac{1}{9}) = \frac{1}{3}$ MINIMUM $(D>0)$

SADDLE POINTS: $(0,0), (0,-1), (1,0)$
MINIMUM: $(\frac{1}{3},-\frac{1}{3})$

7) $f_x = 2x - y + 1 = 0,\ f_y = 2y - x = 0$

$y = 2x + 1,\ 0 = 2(2x+1) - x = 4x + 2 - x,\ x = -\frac{2}{3},\ y = -\frac{1}{3}$

$f_{xx} = 2,\ f_{xy} = -1,\ f_{yy} = 2 \Rightarrow D = 2(-1) - 4 = -6$

LOCAL MINIMUM @ $(\frac{2}{3}, -\frac{1}{3})$ $\quad D<0$

11) $f_x = 4 - 9x^2 - 2y^2 = 0,\ f_y = -4xy$

$x = 0: y = \pm\sqrt{2},\ y = 0: x = \pm\frac{2}{3}$

$f_{xx} = -18x,\ f_{xy} = -4y,\ f_{yy} = -4x$

$f_{xx}(0,\pm\sqrt{2}) = 0,\ f_{xx}(\frac{2}{3},0) = -12,\ f_{xx}(-\frac{2}{3},0) = 12$

$f_{xy}(0,\sqrt{2}) = -4\sqrt{2},\ f_{xy}(0,-\sqrt{2}) = 4\sqrt{2},\ f_{xy}(\pm\frac{2}{3},0) = 0$

$f_{yy}(0,\pm\sqrt{2}) = 0,\ f_{yy}(\frac{2}{3},0) = -\frac{8}{3},\ f_{yy}(-\frac{2}{3},0) = \frac{8}{3}$

$D = f_{xx}f_{yy} - [f_{xy}]^2 = 0 - (4\sqrt{2})^2 = -32$
$= 12(\frac{8}{3}) - 0 = 32$

SADDLE POINTS: $(0,\pm\sqrt{2})$ $\quad D<0$
LOCAL MAXIMUM: $(\frac{2}{3},0)$ $\quad D>0, f_{xx}<0$
MINIMUM: $(-\frac{2}{3},0)$ $\quad D>0, f_{xx}>0$

13) $f_x = 4x^3 - 4y = 0,\ f_y = 4y^3 - 4x = 0$

$y = x^3,\ 4x^9 - 4x = 0,\ x = \pm1,\ y = \pm1$

CRITICAL POINTS: $(0,0), (1,1), (-1,-1)$

$f_{xx} = 12x^2,\ f_{xy} = -4,\ f_{yy} = 12y^2$

$f_{xx}(0,0) = 0,\ f_{xx}(\pm1,\pm1) = 12$

$f_{xy}(0,0) = f_{xy}(\pm1,\pm1) = -4$

$f_{yy}(0,0) = 0,\ f_{yy}(\pm1,\pm1) = 12$

$D = f_{xx}f_{yy} - [f_{xy}]^2 = 0 - 16 = -16$
$= 144 - 16 = 128$

SADDLE POINT: $(0,0)$ $\quad D<0$
LOCAL MINIMA: $(1,1)$ and $(-1,-1)$ $\quad D>0, f_{xx}>0$

17) $f_x = \cos(x+y) + \sin x = 0,\ f_y = \cos(x+y) = 0$

$x + y = \frac{\pi}{2},\ y = \frac{\pi}{2} - x,\ \cos(x + \frac{\pi}{2} - x) + \sin x,\ \sin x = 0,\ x = 0, y = \frac{\pi}{2}$

CRITICAL POINTS: $(j\pi, k\pi + \frac{\pi}{2})$

$f_{xx} = \cos x - \sin(x+y)\ f_{xy} = -\sin(x+y),\ f_{yy} = -\sin(x+y)$

TRIAL POINTS: $(0,\frac{\pi}{2}), (0,\frac{3\pi}{2}), (\pi,\frac{\pi}{2}), (\pi,\frac{3\pi}{2})$

$f_{xx}(0,\frac{\pi}{2}) = 0,\ f_{xx}(0,\frac{3\pi}{2}) = 2,\ f_{xx}(\pi,\frac{\pi}{2}) = -2,\ f_{xx}(\pi,\frac{3\pi}{2}) = -2$

$f_{yy}(0,\frac{\pi}{2}) = -1,\ f_{yy}(0,\frac{3\pi}{2}) = 1,\ f_{yy}(\pi,\frac{\pi}{2}) = 1,\ f_{yy}(\pi,\frac{3\pi}{2}) = -1$

$D = f_{xx}f_{yy} - [f_{xy}]^2 = 0 - 1 = -1$ $\quad D<0$
$= 2 - 1 = 1$ $\quad D>0, f_{xx}>0$
$= -2 - 1 = -3$ $\quad D<0$
$= 2 - 1 = 1$ $\quad D>0, f_{xx}<0$

For $(j\pi, k\pi + \frac{\pi}{2})$
both j+k are even: saddle points
both j+k are odd: local maxima
j even, k odd: local minima
j odd, k even: saddle points

19) $f_x = \frac{1}{x} - 1 = 0,\ f_y = \frac{2}{y} - 4 = 0,\ x = 1, y = \frac{1}{2}$

$f_{xx} = -\frac{1}{x^2},\ f_{xy} = 0,\ f_{yy} = -\frac{2}{y^2}$

$f_{xx}(1,\frac{1}{2}) = -4,\ f_{xy} = 0,\ f_{yy}(1,\frac{1}{2}) = -8$ $\quad D>0, f_{xx}<0$

$D = f_{xx}f_{yy} - [f_{xy}]^2 = -4(-8) = 32 \Rightarrow$ MAXIMUM @ $(1,\frac{1}{2})$

23) $f(x,y) = xe^{y-x^2} + 3ye^{y-x^2}$

$f_x = -2x^2e^{y-x^2} - 6xye^{y-x^2} = 0,\ f_y = xe^{y-x^2} + 3ye^{y-x^2} + 3e^{y-x^2} = 0$

$-2x^2e^{y-x^2} = 6xye^{y-x^2},\ -2x^2 = 6xy,\ y = -\frac{1}{3}x$

$x + 3y + 3 = x + 3(-\frac{1}{3}x) + 3 = 3 = 0???$

29) global max: $f(1,1) = 1+1 = 2$
global min: $f(0,0) = 0+0 = 0$

35) a) $f_x = 1 - 2x - y,\ f_y = 1 - 2y - x,\ x = 1 - 2y,\ x = 1/3$

$0 = 1 - 2(1-2y) - y = 1 - 2 + 4y - y = -1 + 3y,\ y = 1/3$

b) $f'(x,0) = 1 - 2x,\ x = \frac{1}{2}$

$f(0,0) = 0,\ f(2,0) = -2,\ f(\frac{1}{2},0) = \frac{1}{4}$

c) $f(x,2) = x + 2 - x^2 - 4 - 2x = -x^2 - x - 2,\ f'(x,2) = -2x - 1,\ x = -\frac{1}{2}$

$f(0,2) = -2,\ f(2,2) = -8$

$f(0,y) = y - y^2,\ f'(0,y) = 1 - 2y,\ y = 1/2$ (same vals for $f(x,0)$)

$f(2,y) = 2 + y - 4 - y^2 - 2y = -y^2 - y - 2,\ f'(2,y) = -2y - 1,\ y = -\frac{1}{2}$

d) $f(\frac{1}{3},\frac{1}{3}) = \frac{2}{3} - \frac{1}{9} - \frac{1}{9} - \frac{1}{9} = \frac{1}{3} \Rightarrow$ max: $(1/3, 1/3)$